Numer. Math. Theor. Meth. Appl. Vol. **17**, No. 1, pp. 243-274
doi: 10.4208/nmtma.OA-2023-0048 February 2024

An Accurate Numerical Scheme for Mean-Field Forward and Backward SDEs with Jumps

Yabing Sun[1], Jie Yang[2,*] and Weidong Zhao[3]

[1] *College of Science, National University of Defense Technology, Changsha 410073, China*
[2] *School of Mathematics and Statistics, Shandong University, Weihai 264209, China*
[2] *School of Mathematics, Shandong University, Jinan 250100, China*

Received 21 April 2023; Accepted (in revised version) 27 November 2023

Abstract. In this work, we propose an explicit second order scheme for decoupled mean-field forward backward stochastic differential equations with jumps. The stability and the rigorous error estimates are presented, which show that the proposed scheme yields a second order rate of convergence, when the forward mean-field stochastic differential equation is solved by the weak order 2.0 Itô-Taylor scheme. Numerical experiments are carried out to verify the theoretical results.

AMS subject classifications: 65C30, 60H10, 60H35, 65C05

Key words: Mean-field forward backward stochastic differential equation with jumps, stability analysis, error estimates.

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{F}, P)$ be a complete filtered probability space with $\mathbb{F} = \{\mathcal{F}_t\}_{0\le t\le T}$ being the filtration generated by the following two mutually independent stochastic processes:

- The m-dimensional Brownian motion $W = (W_t)_{0\le t\le T}$.
- The Poisson random measure $\{\mu(A\times[0,t]),\ A\in\mathcal{E},\ 0\le t\le T\}$ on $\mathsf{E}\times[0,T]$, where $\mathsf{E} = \mathbb{R}^q\backslash\{0\}$ and $\mathcal{E}$ is its Borel field.

In this paper, we suppose that the Poisson measure μ has the intensity measure

$$\nu(de,dt) = \lambda(de)dt = \lambda F(de)dt,$$

*Corresponding author. *Email addresses:* `sunybly@163.com` (Y. Sun), `jieyang@sdu.edu.cn` (J. Yang), `wdzhao@sdu.edu.cn` (W. Zhao)

 ©2024 Global-Science Press

where $\lambda(de)$ is a Lévy measure on $(\mathsf{E}, \mathcal{E})$ describing the average number of jumps per unit of time, $\lambda = \lambda(\mathsf{E})$ is the intensity of the measure μ and F is the distribution of the jump size e. Here $\lambda(de)$ is a σ-finite measure satisfying

$$\int_{\mathsf{E}} \left(1 \wedge |e|^2\right) \lambda(de) < +\infty.$$

Moreover, we have the compensated Poisson random measure

$$\tilde{\mu}(de, dt) = \mu(de, dt) - \lambda(de)dt,$$

such that $\{\tilde{\mu}(A \times [0,t]) = (\mu - \nu)(A \times [0,t])\}_{0 \leq t \leq T}$ is a martingale for any $A \in \mathcal{E}$.

The Poisson measure μ can generate a sequence of pairs $(\tau_i, e_i), i = 1, 2, \ldots, N_T$ with $\tau_i \in [0,T], i = 1, 2, \ldots, N_T$, representing the jump times of N_t and $e_i \in \mathsf{E}, i = 1, 2, \ldots, N_T$ the corresponding jump sizes satisfying $e_i \overset{iid}{\sim} F$. Here $N_t = \mu(\mathsf{E} \times [0,t])$ is a Poisson process with intensity λ, which counts the number of jumps of μ occurring in $[0,t]$. For more details of the Poisson random measure and Lévy measure, the readers are referred to [6, 17].

We are interested in the following general mean-field forward backward stochastic differential equations with jumps (MFBSDEJs for short) on $(\Omega, \mathcal{F}, \mathbb{F}, P)$

$$\begin{aligned} X_t^{0,X_0} &= X_0 + \int_0^t \mathbb{E}\big[b\big(s, X_s^{0,x_0}, x\big)\big]\big|_{x = X_s^{0,X_0}} ds \\ &\quad + \int_0^t \mathbb{E}\big[\sigma\big(s, X_s^{0,x_0}, x\big)\big]\big|_{x = X_s^{0,X_0}} dW_s \\ &\quad + \int_0^t \int_{\mathsf{E}} \mathbb{E}\big[c\big(s, X_{s-}^{0,x_0}, x, e\big)\big]\big|_{x = X_{s-}^{0,X_0}} \tilde{\mu}(de, ds), \\ Y_t^{0,X_0} &= \mathbb{E}\big[\Phi\big(X_T^{0,x_0}, x\big)\big]\big|_{x = X_T^{0,X_0}} \\ &\quad + \int_t^T \mathbb{E}\big[f\big(s, \Theta_s^{0,x_0}, \theta\big)\big]\big|_{\theta = \Theta_s^{0,X_0}} ds \\ &\quad - \int_t^T Z_s^{0,X_0} dW_s - \int_t^T \int_{\mathsf{E}} U_s^{0,X_0}(e) \tilde{\mu}(de, ds), \end{aligned} \tag{1.1}$$

where

$$\Theta_s^{0,x} = \left(X_s^{0,x}, Y_s^{0,x}, Z_s^{0,x}, \Gamma_s^{0,x}\right)$$

with $x = x_0$ and X_0 being the initial values of mean-field forward stochastic differential equations with jumps (MSDEJs). Here, $\Gamma_s^{0,x}$ is defined by

$$\Gamma_s^{0,x} = \int_{\mathsf{E}} U_s^{0,x}(e) \eta(e) \lambda(de)$$

for a given function $\eta : \mathsf{E} \to \mathbb{R}$ satisfying $\sup_{e \in \mathsf{E}} |\eta(e)| < +\infty$,

$$\begin{aligned} &b : [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^d, \\ &\sigma : [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \to \mathbb{R}^{d \times m}, \\ &c : [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \times \mathsf{E} \to \mathbb{R}^d \end{aligned}$$