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SIMULATION OF CEREBRAL INFUSION TESTS USING A POROELASTIC MODEL

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Abstract. In an infusion test the apparent rate of cerebrospinal fluid (CSF) production is temporarily increased through injection of fluid directly into the CSF system with the result that CSF pressure rises, in theory to a new plateau average, and the change in pressure level gives a measure of resistance to CSF outflow and the rate of approach to the plateau gives information about cerebral compliance. In the first part of this paper we give details of a two-fluid (blood and CSF) spherically symmetric poroelastic model that can simulate an infusion test which includes oscillations in blood pressure. This model has been applied to clinical data where the infusion rate and arterial blood pressure are input and an oscillatory CSF pressure is computed along with spatial parenchyma displacement, strain and local changes in CSF content. In the later part of this paper, the poroelastic model is simplified by spatial integration resulting in a one-compartment model that includes blood pressure oscillations but which, when they are ignored, reduces to a well known one-compartment model. When the arterial pressure pulsations are included, their interaction with a non-linear compliance results in solutions that have to be interpreted very carefully to predict parameter values.

Key words. infusion test, intracranial cerebrospinal fluid pressure, poroelasticity

1. Introduction

The infusion test can be used to aid interpretation of CSF function: for a short period of time the rate of CSF production is increased and the change in CSF pressure is measured. Usually the time average value rises to a new plateau value, the rate of rise giving information about the cerebral compliance and the plateau value showing the resistance to CSF absorption, two important clincial indicators of CSF function. The simplest interpretation of this test assumes that the CSF is contained in a single compliant compartment so that the pressure variation is described by a first order ODE in time with CSF production as input parameter. Such models do not take arterial pressure fluctutation into account and the CSF pressure calculated, while slowly varying in time, does not fluctuate on the scale of arterial pressure pulsations, see for example [8] for a review of such models.

In a series of papers ([9], [10], [13]) a poroelastic model was developed for predicting changes in cerbrospinal fluid pressure in a number of situations, originally for obstructive hydrocephalus and then extended to some time dependent situations. The original model was based on a long time scale so fluctuations in arterial pressure were neglected, indeed that model considered the brain only as a two-phase material with a porous elastic phase through which CSF could move and where changes in CSF pressure (intracranial pressure, ICP) were coupled to changes in stress and strain in the elastic phase. In a more recent paper, [15], a mathematical model was developed that included multiple fluid phases, in individual compartments, separated from each other and where CSF was one of the fluids. Here we develop that model for two fluid compartments, retaining a continuum hypothesis and treating the brain as having three compartments or phases: a porous elastic

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compartment, a CSF compartment and a single blood compartment. There is no exchange of fluid between the CSF and blood compartments. Both the elastic and CSF compartments have spatial, as well as temporal, dependence. The model retains spherical symmetry and so is the simplest spatially varying complete model for ICP fluctuations, complete in the sense that the model can be used to simulate changes in ICP in for example, an infusion test. However the spatial dependence means that the model is time consuming to solve numerically. In this paper we set out the spatially varying model and then derive from that model, a spatially averaged model that while still requiring numerical solution, can be integrated rapidly.

2. Spatially varying model

2.1. Biot's theory of poroelastic deformation. In order to derive a three-phase poroelastic model for the brain we begin by briefly reviewing the theory of [2], particularly using the notation described in [11] (see *pp*17-21 therein). For a fluid-filled, porous, solid matrix, Biot supposed a continuum description with a strain ε, overall stress σ, fluid pressure p, and an additional variable ζ, the increment in fluid content per volume element. Assuming a physical state to be locally described by pressure and stress and linearising the relations between $\varepsilon, \sigma, p, \zeta$ gives that

$$\varepsilon = a_{11}\sigma + a_{12}p, \tag{2.1}$$

$$\zeta = a_{21}\sigma + a_{22}p. \tag{2.2}$$

Furthermore, Biot assumed the existence of an energy density

$$U = \sigma\varepsilon + \zeta p, \tag{2.3}$$

so that the condition $\frac{\partial^2 U}{\partial\zeta\partial\varepsilon} = \frac{\partial^2 U}{\partial\varepsilon\partial\zeta}$, implies $a_{12} = a_{21}$. Letting $a_{11} = 1/K$, where K is the bulk modulus of the elastic phase, $a_{12} = a_{21} = \alpha/K$, with α the Biot–Willis parameter, and $a_{22} = \alpha/(\beta K)$, with β Skempton's coefficient, gives on rearranging (2.1) and (2.2),

$$\sigma = K\varepsilon - \alpha p, \tag{2.4}$$

$$\zeta = \alpha\varepsilon + \frac{\alpha(1-\alpha\beta)}{\beta K}p. \tag{2.5}$$

The system is completed by assuming that the strain is a result of a displacement u and that fluid flow through the porous matrix obeys a Darcy flow model so that the balance of momentum and the conservation of fluid give, if u is the matrix displacement (and neglecting acceleration of fluid through the matrix),

$$\rho\frac{\partial^2 u}{\partial t^2} = \nabla\cdot\sigma, \tag{2.6}$$

$$\frac{\partial\zeta}{\partial t} = \nabla\cdot\frac{k}{\mu}\nabla p, \tag{2.7}$$

where ρ is the density of the solid-fluid continuum, k is a permeability, and μ the fluid viscosity. In the model of the brain used in [9], [10], [13], the solid matrix represented the brain parenchyma and the fluid the CSF. In these models, the time scale was long enough that the left hand side of both these equations was neglected, and the time dependence only entered the model in a quasi-stationary manner through a boundary condition that expressed conservation of CSF.