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Non-local Problem for the Loaded Integral-differential Equation in Double-connected Domain

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Abstract. In this work an existence and uniqueness of solution of the non-local boundary value problem for the loaded elliptic-hyperbolic type equation with integral-differential operations in double-connected domain have been investigated. The uniqueness of solution is proved by the method of integral energy using an extremum principle for the mixed type equations, and the existence is proved by the method of integral equations.

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1 Introduction and formulation of a problem

We note that with intensive research on problem of optimal control of the agroeconomical system, regulating the label of ground waters and soil moisture, it has become necessary to investigate a new class of equations called "LOADED EQUATIONS". For the first time it was given the most general definition of a Loaded equations and various Loaded equations are classified in detail by A.M.Nakhushev [1]. After this work very interesting results on the theory of boundary value problems for the loaded equations of parabolic, parabolic-hyperbolic and elliptic-hyperbolic types were published, for example, see [2–4] and [5]. In this direction, we studied some local and non-local problems for the loaded second and third order elliptic-hyperbolic type equations in double-connected domains

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(see [6–9]). Notice that non-local problems for the loaded integral-differential elliptic-hyperbolic type equations in double-connected domains have not been investigated. In the given paper, for the equation:

$$u_{xx}+\mathrm{sgn}y u_{yy}+\frac{1-\mathrm{sgn}y}{2}\sum_{k=1}^{n}R_k(x,0)=0 \tag{1.1}$$

with operators [10]:

$$R_k(x,y)=R_k\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2}\right)=\begin{cases} p_k(\xi)D^{\alpha_k}_{\xi 1}u\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2}\right), & \text{at } q\le x\le 1,\\ r_k(\eta)D^{\beta_k}_{-1\eta}u\left(\frac{\xi+\eta}{2},\frac{\xi-\eta}{2}\right), & \text{at } -1\le x\le -q, \end{cases} \tag{1.2}$$

$$D^{\alpha_k}_{ax}f(x)=\frac{1}{\Gamma(-\alpha_k)}\int_a^x (x-t)^{-1-\alpha_k}f(t)\mathrm{d}t, \qquad -1<\alpha_k<0, \tag{1.3}$$

$$D^{\beta_k}_{xa}f(x)=\frac{1}{\Gamma(-\alpha_k)}\int_x^a (t-x)^{-1-\beta_k}f(t)\mathrm{d}t, \qquad -1<\beta_k<0, \tag{1.4}$$

we will investigate the uniqueness and the existence of solution of the non-local problem. Let's, Ω, be double connected domain, bounded with two lines:

$$\sigma_1: x^2+y^2=1;\ \ \sigma_2: x^2+y^2=q^2,\ y>0,$$

and characteristics:

$$A_jC_1: x+(-1)^j y=(-1)^{j+1};\ B_jC_2: x+(-1)^j y=(-1)^{j+1}\cdot q;\ (0<q<1),\ (j=1,2)$$

of the Eq. (1.1) at $y<0$, where $x+y=\xi$, $x-y=\eta$; $A_1(1;0)$, $A_2(-1;0)$, $B_1(q;0)$, $B_2(-q;0)$, $C_1(0;-1)$, $C_2(0;-q)$.

Introduce designations: $\theta_1(x)=\frac{x+1}{2}+i\cdot\frac{x-1}{2}$, $\theta_2(x)=\frac{x-1}{2}-i\cdot\frac{x+1}{2}$, $(i^2=-1)$;

$$\Omega_0=\Omega\cap(y>0), \qquad \Delta_1=\Omega\cap(x+y>q)\cap(y<0), \qquad \Delta_2=\Omega\cap(y-x>q)\cap(y<0);$$
$$D_1=\Omega\cap(-q<x+y<q)\cap(x>0), \qquad D_2=\Omega\cap(-q<y-x<q)\cap(x<0);$$
$$D_3=\Omega\cap(-1<x+y<-q)\cap(-1<y-x<-q), \qquad D_0=\Omega_0\cup\Delta_1\cup\Delta_2;$$
$$I_{2+j}=\left\{x: 0<(-1)^{j-1}x<q\right\}, \qquad I_j=\left\{x: \frac{q+1}{2}<(-1)^{j-1}x<1\right\}, \qquad (j=1,2).$$

In the domain Ω the following problem is investigated.

Problem I. To find a function $u(x,y)$ satisfies the following properties:

1) $u(x,y)\in C(\bar{\Omega})$;

2) $u(x,y)$ is a regular solution of the Eq. (1.1) in the domain of $\Omega\backslash(y-x=\pm q)\backslash(x+y=\pm q)$, besides, $u_y\in C(A_1B_1\cup A_2B_2)$ and $u_y(x,0)$ can tend to infinity an order of less unit at $x\to\pm q$, and finite at $x\to\pm 1$;