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Conservation Laws for CKdV and BSSK Systems

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Abstract. In current paper, the coupled KdV (CKdV) system and Bosonized Supersymmetric Sawada-Kotera (BSSK) system are considered. Some linearly independent conservation laws for the two systems are derived via the first homotopy approach and symbolic computation.

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1 Introduction

It is well known that one of the important problems in the fields of mathematics and physics is to construct as many as possible conservation laws of nonlinear differential system (NLDS). Conservation laws describe essential physical properties of the modeled process. And in the study of differential system, especially integrable system and soliton theory, conservation laws play a key role. Conservation laws have some applications in finding exact solutions, analyzing various characteristics of solutions, and discussing the qualitative properties such as the bi- or tri-Hamiltonian structures, Liouville integrability, recursion operators and so forth. A sequence of methods, such as Nöether's theorem [1], multiplier method [2], the first homotopy method [3, 4], Lax pair method [5] and others, have been well established and used. However, among these methods, the first homotopy method has its advantage in dealing with complicated forms of multipliers or equations by using homotopy operators arising from differential geometry.

We begin in Section 2 by listing some definitions and theorems related with the first homotopy method. In Section 3 and Section 4, we apply the first homotopy method

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to construct some linearly independent conservation laws of CKdV and BSSK systems respectively. Finally, a short summary is given.

2 Preliminaries

Conservation laws for a NLDS of order k,

$$\Gamma^{\alpha}(\mathrm{x}, u, \partial u, \cdots, \partial^k u) = 0, \quad \alpha = 1, \cdots, N, \tag{2.1}$$

with independent variables $\mathrm{x} \equiv (x_1, x_2, \cdots, x_n)$ and dependent variables $u \equiv (u^1, u^2, \cdots, u^m)$, are written by scalar divergence expressions

$$D_{x_1}\Phi^{x_1} + D_{x_2}\Phi^{x_2} + \cdots + D_{x_n}\Phi^{x_n} = 0,$$

where Φ^{x_i} $(i = 1, 2, \cdots, n)$ represent fluxes and total derivative operators

$$D_{x_i} = \frac{\partial}{\partial x_i} + u_i^{\mu}\frac{\partial}{\partial u^{\mu}} + u_{ii_1}^{\mu}\frac{\partial}{\partial u_{i_1}^{\mu}} + u_{ii_1i_2}^{\mu}\frac{\partial}{\partial u_{i_1i_2}^{\mu}} + \cdots, \quad \mu = 1, \cdots, m.$$

In order to determine some conservation laws for system (2.1), one firstly needs to find multipliers $\{\Lambda_\alpha = \Lambda_\alpha(\mathrm{x}, U, \partial U, \cdots, \partial^l U)\}_{\alpha=1}^N$, such that a linear combination of equations is a divergence expression, i.e.,

$$\Lambda_\alpha \Gamma^{\alpha}(\mathrm{x}, U, \partial U, \cdots, \partial^k U) = D_{x_1}\Phi^{x_1} + D_{x_2}\Phi^{x_2} + \cdots + D_{x_n}\Phi^{x_n},$$

which holds identically for arbitrary functions $U(\mathrm{x})$. The following definitions and theorems (see [3, 4]) are necessary.

Theorem 2.1. *A set of non-singular multipliers* $\{\Lambda_\alpha = \Lambda_\alpha(\mathrm{x}, U, \partial U, \cdots, \partial^l U)\}_{\alpha=1}^N$ *yields a local conservation law of system* (2.1) *if and only if the identities*

$$E_{U^j}(\Lambda_\alpha \Gamma^{\alpha}(\mathrm{x}, U, \partial U, \cdots, \partial^k U)) = 0,$$

hold for arbitrary functions $U(\mathrm{x})$.

Definition 2.1. *The n-dimensional Euler operator with respect to* $U(x_1, x_2, \cdots, x_n)$ *is given by*

$$E_U^{(l_1, l_2, \cdots, l_n)} = \sum_{k_1=l_1}^{\infty} \cdots \sum_{k_n=l_n}^{\infty} \binom{k_1}{l_1} \cdots \binom{k_n}{l_n} D_{x_1}^{k_1-l_1} \cdots D_{x_n}^{k_n-l_n} \frac{\partial}{\partial U^{(k_1+k_2+\cdots+k_n)}},$$

where $U^{(k_1+k_2+\cdots+k_n)} = \partial^{k_1+k_2+\cdots+k_n} U / \partial^{k_1} x_1 \partial^{k_2} x_2 \cdots \partial^{k_n} x_n$.