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Global Well-Posedness of Classical Solutions with Large Initial Data to the Two-Dimensional Isentropic Compressible Navier-Stokes Equations

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Abstract. We establish the global existence and uniqueness of classical solutions to the Cauchy problem for the two-dimensional isentropic compressible Navier-Stokes equations with smooth initial data under the assumption that the viscosity coefficient μ is large enough. Here we do not require that the initial data is small.

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1 Introduction

In this paper, we consider the following compressible Navier-Stokes equations in $\mathbb{R}^2$

$$\begin{cases}\rho_t+\mathrm{div}(\rho u)=0,\\ (\rho u)_t+\mathrm{div}(\rho u\otimes u)-\mu\triangle u-(\mu+\lambda)\nabla(\mathrm{div}u)+\nabla P(\rho)=0,\end{cases}\tag{1.1}$$

where ρ, $u=(u^1,u^2)$ and $P=A\rho^\gamma$ $(A>0,\gamma>1)$ are the fluid density, velocity and pressure, respectively. The constant viscosity coefficients μ and λ satisfy the physical restrictions:

$$\mu>0,\quad \mu+\lambda\geq 0.\tag{1.2}$$

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Let $\tilde{\rho}$ be a fixed positive constant. We look for the solutions, $(\rho(x,t),u(x,t))$, to the Cauchy problem for (1.1) with the far field behavior:

$$u(x,t)\to 0,\quad \rho(x,t)\to\tilde{\rho}>0,\quad \text{as } |x|\to\infty, \tag{1.3}$$

and initial data,

$$(\rho,u)|_{t=0}=(\rho_0,u_0),\quad x\in\mathbb{R}^2. \tag{1.4}$$

There are huge literature on the large time existence and behavior of solutions to (1.1). The one-dimensional problem has been studied by many people, see [1–4] and the references therein. For the multi-dimensional case, the local existence and uniqueness of classical solutions are known in the absence of vacuum (see [5,6]), for strong solutions and the case that the initial density need not be positive and may vanish in an open sets see [7–10]. The global classical solutions were first obtained by Matsumura-Nishida [11] for the initial data close to a non-vacuum equilibrium in the sense of some Sobolev space H^s. Later, Hoff [12,13] studied the problem for discontinuous initial data. The existence of weak solutions with arbitrary initial data was studied (the far field is vacuum, that is $\tilde{\rho}=0$) by Lions [14] (see also Feireisl [15]), where he obtains the global existence of weak solutions-defined as solutions with finite energy when the exponent γ is suitably large. The main restriction on initial data is that the initial energy is finite, so that the density vanishes at far fields, or even has compact support. Zhang and Fang established the existence of the global weak solutions in $\mathbb{R}^2$ with the small initial energy of the initial data and the initial density bounded away from zero (see [16]). Recently, Huang-Li-Xin [17] establish the existence of the global classical solutions to the Cauchy problem for the compressible Navier-Stokes equations in 3-D with smooth initial data which are small energy. By the inspiration of [17], we establish the global existence of classical solutions to the Cauchy problem to the 3-D compressible Navier-Stokes equations with general initial data under the assumption that the viscosity coefficient μ is large enough (see [18]).

Note that the Gagliardo-Nirenberg-Sobolev inequality in $\mathbb{R}^2$ is different from the Gagliardo-Nirenberg-Sobolev inequality in $\mathbb{R}^3$, for example, $\|u\|_{L^6}\le C\|\nabla u\|_{L^2}$ in $\mathbb{R}^3$ which is a base of the proof in [18], but it is not correct in $\mathbb{R}^2$. So the result in [18] can not include the two-dimensional case. In this paper we generalized the result in [18] to the two-dimensional case, to study the global existence of classical solutions to Cauchy problem (1.1) for general initial data. We obtain the well-posedness of global classical solutions for large initial data, under the assumption that the viscosity coefficient μ is large enough. Here we do not require that the initial energy is small. In this paper some new difficulties are overcame.

Before stating the main results, we explain the notations and conventions used throughout this paper. We denote

$$\int f\,\mathrm{d}x=\int_{\mathbb{R}^2} f\,\mathrm{d}x.$$

For $1\le r\le\infty$, we denote the standard homogeneous and inhomogeneous Sobolev