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Global Existence and Asymptotic Behavior for a Coupled System of Viscoelastic Wave Equations with a Delay Term

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Abstract. We are interested in the study of a coupled system of viscoelastic wave equations with a delay term. Firstly global existence of the solutions is proved. The asymptotic behavior is obtained by using multiplier technique proved by A. Guessmia [1], however in the unstable set for certain initial data bolstered with some conditions, we obtain the blow up of the solutions for various sign of initial energy negative, positive or null. We use the same technique in [2] with a necessary modification imposed by our problem. This improves the earlier results in the literature

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1 Introduction

In this paper, we consider the following problem :

$$u_{tt}-\Delta u-\Delta u_{tt}+\int_0^t g_1(t-s)\Delta u(s)\mathrm{d}s+\sum_{i=1}^{2}\mu_i u_t(x,t-\tau(i))+f_1(u,v)=0, \tag{1.1}$$

$$v_{tt}-\Delta v-\Delta v_{tt}+\int_0^t g_2(t-s)\Delta v(s)\mathrm{d}s+\sum_{i=1}^{2}\alpha_i v_t(x,t-\tau(i))+f_2(u,v)=0, \tag{1.2}$$

$$u(x,t)=0,\ v(x,t)=0 \qquad \text{on } \Gamma\times(0,+\infty), \tag{1.3}$$

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$$u(x,0)=u_0(x),\ v(x,0)=v_0(x),\ u_t(x,0)=u_1(x),\ x\in\Omega, \tag{1.4}$$
$$u_t(x,t-\tau(2))=\phi_0(x,t-\tau(2)),\ x\in\Omega,\ t\in(0,\tau_2), \tag{1.5}$$
$$v_t(x,t-\tau(2))=\phi_1(x,t-\tau(2)),\ x\in\Omega,\ t\in(0,\tau_2),\ \tau(1)=0,\ \tau(2)=\tau_2, \tag{1.6}$$

where Ω is a bounded domain in $\mathbb{R}^n, n\in\mathbb{N}^*$, with a smooth boundary $\partial\Omega$, and $g_1,g_2:\mathbb{R}^+\to\mathbb{R}^+$, $f_i(\cdot,\cdot):\mathbb{R}^2\to\mathbb{R}$, $i=1,2$, are given functions which will be specified later, $\tau_2>0$ is a time delay, where $\mu_1,\alpha_1,\alpha_2,\mu_2$ are positive real numbers and the initial data (u_0,u_1,f_0), (v_0,v_1,f_1) belonging to a suitable space. Problems of this type arise in material science and physics.

Recently, the authors of [3] considered the following coupled system of quasilinear viscoelastic equation in canonical form without delay terms

$$\begin{cases} |u_t|^\rho u_{tt}-\Delta u-\gamma_1\Delta u_{tt}+\displaystyle\int_0^t g_1(t-s)\Delta u(s)\mathrm{d}s+f_1(x,u)=0, & \text{in } \Omega\times(0,+\infty),\\ |v_t|^\rho v_{tt}-\Delta v-\gamma_2\Delta v_{tt}+\displaystyle\int_0^t g_2(t-s)\Delta v(s)\mathrm{d}s+f_2(x,u)=0, & \text{in } \Omega\times(0,+\infty), \end{cases} \tag{1.7}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(n\geq 1)$with a smooth boundary $\partial\Omega,\gamma_1,\gamma_2\geq 0$ are constants and ρ is a real number such that $0<\rho<2/(n-2)$ if $n\geq 3$ or $\rho>0$ if $n=1,2$. The functions u_0,u_1,v_0 and v_1 are given initial data. The relaxations functions g_1 and g_2 are continuous functions and $f_1(u,v),f_2(u,v)$ represent the nonlinear terms. The authors proved the energy decay result using the perturbed energy method.

Many authors considered the initial boundary value problem as follows

$$\begin{cases} u_{tt}-\Delta u+\displaystyle\int_0^t g_1(t-s)\Delta u(s)\mathrm{d}s+h_1(u_t)=f_1(x,u), & \text{in } \Omega\times(0,+\infty),\\ v_{tt}-\Delta v+\displaystyle\int_0^t g_2(t-s)\Delta v(s)\mathrm{d}s+h_2(v_t)=f_2(x,u), & \text{in } \Omega\times(0,+\infty), \end{cases} \tag{1.8}$$

when the viscoelastic terms g_i $(i=1,2)$ are not taken into account in (1.8), Agre and Rammaha [4] obtained several results related to local and global existence of a weak solution. By using the same technique as in [5], they showed that any weak solution blow-up in finite time with negative initial energy. Later Said-Houari [6] extended this blow up result to positive initial energy. Conversely, in the presence of the memory term $(g_i\neq 0$ $(i=1,2.)$, there are numerous results related to the asymptotic behavior and blow up of solutions of viscoelastic systems. For example, Liang and Gao [7] studied problem (1.8) with $h_1(u_t)=-\Delta u_t$, $h_2(v_t)=-\Delta u_t$. They obtained that, under suitable conditions on the functions $g_i,f_i,i=1,2$, and certain initial data in the stable set, the decay rate of the energy functions is exponential. On the contrary, for certain initial data in the unstable set, there are solutions with positive initial energy that blow-up in finite time. For $h_1(u_t)=|u_t|^{m-1}u_t$ and $h_2(v_t)=|v_t|^{r-1}v_t$. Hun and Wang [8] established several results related to local existence, global existence and finite time blow-up (the initial energy $E(0)<0$).