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Mild Solution of Stochastic Equations with Lèvy Jumps: Existence, Uniqueness, Regularity and Stability

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Abstract. The existence and uniqueness of mild solution to stochastic equations with jumps are established, a stochastic Fubini theorem and a type of Burkholder-Davis-Gundy inequality are proved, and the two formulas are used to study the regularity property of the mild solution of a general stochastic evolution equation perturbed by Lévy process. Then the authors prove the moment exponential stability, almost sure exponential stability and comparison principles of the mild solution. As applications, the stability and comparison principles of stochastic heat equation with Lévy jump are given.

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1 Introduction

In recent years, there are so many monographs concerning stochastic partial differential equations with Lévy jump and it's applications in physics, economics, statistical mechanics, fluid dynamics and finance etc. For these theory and applications, one can see [1–3] and references therein. In this article, the existence, uniqueness, regularity and stability for the mild solution of the stochastic partial differential equations with Lévy jump

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are studied. There are a lot of works dealing with existence and uniqueness for stochastic partial differential equations with jump process. In [4], existence and uniqueness for solutions of stochastic reaction diffusion equations driven by Poisson random measures are obtained. In [5], Malliavin calculus is applied to study the absolute continuity of the law of the solutions of stochastic reaction diffusion equations driven by Poisson random measures. In [6], a minimal solution is obtained for the stochastic heat equation driven by non-negative Lévy noise with coefficients of polynomial growth. In [7], a weak solution is established for the stochastic heat equation driven by stable noise with coefficients of polynomial growth. In [8], existence and uniqueness for solutions of stochastic generalized porous media equations with Lévy jump are obtained.

For stability of stochastic partial differential equations, there are also lots of works. One can find the related topics in [9–11] and reference therein. The stability of a linear equation with jump coefficient is studied in [11]. In [12], the stability of a semilinear stochastic differential equation with Wiener process is investigated. The exponential stability of general nonlinear stochastic differential equations with Wiener processes is studied in [13], and the asymptotic and exponential stability of the nonlinear stochastic delay differential equations driven by Wiener processes are considered in [14] and [15] respectively. In [16], stability of infinite dimensional stochastic evolution equations with memory and Lévy jumps is studied.

The main aim of this paper is to study existence, uniqueness, regularity and stability of stochastic equation:

$$\mathrm{d}X(t)=[A(X(t))+B(X(t))]\mathrm{d}t+Q(X(t))\mathrm{d}W(t)+\int_Z F(X(t-),x)\widetilde{N}(\mathrm{d}t,\mathrm{d}x). \tag{1.1}$$

In [16], the intensity measure λ of $\widetilde{N}(\mathrm{d}t,\mathrm{d}x)$ is finite, while the intensity measure in this article is σ-finite, and also the classical Lipschitz condition (2.5) in [16] is relaxed to condition A.5 in this article. The authors of this article prove the existence and uniqueness of mild solution of (1.1), the continuity of the solution with respect to initial data. And then they prove the stochastic Fubini theorem for compensated Poisson random measure whose intensity measure is σ-finite compared to finite case in [16]. Furthermore a new type of Burkholder-Davis-Gundy inequality which is more precise than Lemma 2.2 in [16] is got. Using the two basic tools, the authors get the regularity property of mild solution of (1.1) without conditions (2.15) and (2.16) in [16] which are critical there. Then the authors also prove the almost sure exponential stability without condition (2.28) in [16].

For the stability of stochastic differential equations with Wiener noise, it has been deeply studied, see [9–14,16], and reference therein. In this article, the stability of stochastic equations with Lévy noise is considered. This will bring various difficulties from calculus and probability compared to studying stochastic equations with Wiener noise. By using method from [16] and [17], the authors derive some sufficient conditions to ensure stability of infinite dimensional stochastic systems in sense of both moment exponential stability and almost sure exponential stability, and in the following examples are given to illustrate the two kinds of stabilities. Finally, the authors compare the stability between