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A Generalized (G'/G)-Expansion Method to Find the Traveling Wave Solutions of Nonlinear Evolution Equations

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Abstract. In this article, we construct the exact traveling wave solutions for nonlinear evolution equations in the mathematical physics via the modified Kawahara equation, the nonlinear coupled KdV equations and the classical Boussinesq equations, by using a generalized (G'/G)-expansion method, where G satisfies the Jacobi elliptic equation. Many exact solutions in terms of Jacobi elliptic functions are obtained.

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1 Introduction

The investigation of the exact solutions for nonlinear evolution equations plays an important role in the study of soliton theory. In recent years, many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform method [1], the Hirota method [2], the Backlund transform method [3], the exp- function method [4], truncated Painleve expansion method [5], the Weierstrass elliptic function method [6], the tanh- function method [7] and the Jacobi elliptic function expansion method [8,9]. There are other methods which can be found in [10,11].

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Wang *et al.* [12] have introduced a simple method which is called, the $(G^{'}/G)$- expansion method to look for traveling wave solutions of nonlinear evolution equations, where $G=G(\xi)$ satisfies the second order linear ordinary differential equation $G^{''}(\xi)+\lambda G'(\xi)+\mu G(\xi)=0$ and λ μ are arbitrary constants. For further references, see the articles [13, 14]. Recently, Zayed [15] introduced an alternative approach, which is called a generalized $(\frac{G^{'}}{G})$- expansion method . The main idea of this alternative approach is that the traveling wave solutions of nonlinear differential equations can be expressed by a polynomial in $(G^{'}/G)$,where $G=G(\xi)$ satisfies the Jacobi elliptic equation $[G'(\xi)]^2=e_2G^4(\xi)+e_1G^2(\xi)+e_0$, $\xi=x-Vt$ and e_2, e_1, e_0, V are arbitrary constants while $'=\mathrm{d}/\mathrm{d}\xi$. The objective of this article is to apply the generalized $(G^{'}/G)$-expansion method to construct the traveling wave solutions for nonlinear evolution equations in the mathematical physics via the modified Kawahara equation, the coupled KdV equations and the classical Boussinesq equations, in terms of the Jacobi elliptic functions.

2 Description of a generalized $(G^{'}/G)$-expansion method

Suppose we have the following nonlinear partial differential equation

$$F(u,u_t,u_x,u_{tt},u_{xt},u_{xx},\cdots)=0, \tag{2.1}$$

where $u=u(x,t)$ is an unknown function, F is a polynomial in $u(x,t)$ and its various partial derivatives, in which the highest order derivatives and the nonlinear terms are involved. In the following we give the main steps of a generalized $(G^{'}/G)$-expansion method [15]:

Step 1. We start with, the traveling wave variable

$$u(x,t)=u(\xi), \qquad \xi=x-Vt, \tag{2.2}$$

where V is a constant which, permits us reducing Eq. (2.1) to an ODE for $u=u(\xi)$ in the form

$$P(u,u',u'',u''',\cdots)=0. \tag{2.3}$$

Step 2. Suppose the solution of Eq. (2.3) can be expressed by a polynomial in $(G^{'}/G)$ as follows

$$u(\xi)=\sum_{i=0}^{n}\alpha_i\left(\frac{G^{'}}{G}\right)^i, \tag{2.4}$$

where $G=G(\xi)$ satisfies the following Jacobi elliptic equation:

$$[G'(\xi)]^2=e_2G^4(\xi)+e_1G^2(\xi)+e_0, \tag{2.5}$$

where α_i ,e_2,e_1,e_0 and V are arbitrary constants to be determined later provided $\alpha_n\neq 0$. The positive integer "n" can be determined by considering the homogeneous balance