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Morrey Estimates for Nondivergence Parabolic Operators with Discontinuous Coefficients on Homogeneous Groups

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Abstract. In this paper, by establishing the boundedness of singular integral operators with variable kernels and their commutators with BMO functions on Morrey spaces of homogeneous groups, we prove a local a prior estimate in Sobolev-Morrey space for solutions to the nondivergence parabolic equation with discontinuous coefficients.

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1 Introduction

In recent years, there has been extensive study on a priori estimates for partial differential operators by using singular integral theory (see [1–3]). For example, authors in [4–7] proved Morrey estimates for nondivergence elliptic operators with discontinuous coefficients on Euclidean spaces. Bramanti and Brandolini proved Schauder estimates for parabolic operators of Hörmander type in [8] and L^p-estimates for hypoelliptic operators of Hörmander type in [9], respectively. The L^p-estimates and Morrey estimates for ultraparabolic operator of Kolmogorov-Fokker-Planck type

$$\mathcal{L}=\sum_{i,j=1}^{q} a_{ij}(x,t)\partial_{x_i}\partial_{x_j}+\sum_{i,j=1}^{N} x_i b_{ij}\partial_{x_j}-\partial_t, \quad (x,t)\in\mathbb{R}^{N+1} \tag{1.1}$$

were considered in [10] and [11], respectively, where the coefficients $a_{ij}\in VMO\cap L^\infty(\Omega)$, $\{b_{ij}\}$ is a constant real matrix with a suitable upper triangular structure. The class of

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operators (1.1) contains prototypes of the Fokker-Planck operators describing Brownian motions of a particle in fluid, as well as Kolmogorov operators depicting systems with $2n$ degrees of freedom (see [12]).

Let $a_{ij}=\delta_{ij}$, $X_i=\partial_{x_i}$, and $X_0=\sum_{i,j=1}^{N} x_i b_{ij}\partial_{x_j}-\partial_t$. Then (1.1) becomes a hypoelliptic operator

$$\mathcal{L}=\sum_{i,j=1}^{q} X_i^2+X_0,$$

where $X_0,X_1,\cdots,X_q$ satisfy the Hörmander condition, i.e., the Lie algebra generated at every point by the fields $X_0,X_1,\cdots,X_q$ is $\mathbb{R}^{N+1}$. In [12], the L^p-estimate for the following operator on homogeneous groups

$$\mathcal{L}=\sum_{i,j=1}^{q} a_{ij}X_iX_j+a_0X_0,$$

was proved, where a_0, $a_{ij}\in VMO\cap L^\infty(\Omega)$ and vector fields X_0, X_1, $\cdots,X_q$ satisfy the Hörmander condition.

The aim of this paper is to establish Morrey estimates for nondivergence parabolic operators with discontinuous coefficients and lower order terms on homogeneous groups

$$\mathbf{H}=\sum_{i,j=1}^{q} a_{ij}(z)X_iX_j+\sum_{i=1}^{q} b_i(z)X_i+c(z)-\partial_t, \quad z=(x,t)\in\mathbb{R}^{N+1}, \tag{1.2}$$

where $X_1,\cdots,X_q$ are the first layer of the basis of vector fields of homogeneous groups, the coefficients a_{ij},b_i,c satisfy the following several assumptions:

• (H1) uniform ellipticity condition: $a_{ij}(z)\in L^\infty(\Omega)$, $\Omega\subset\mathbb{R}^{N+1}$ and there exists $\mu>0$ such that

$$\frac{1}{\mu}\sum_{j}^{q}\zeta_j^2\le\sum_{i,j}^{q}a_{ij}(z)\zeta_i\zeta_j\le\mu\sum_{j}^{q}\zeta_j^2, \quad \forall(\zeta_1,\cdots,\zeta_q)\in\mathbb{R}^q.$$

• (H2) very weak regularity condition: $a_{ij}(z)\in VMO(\Omega)$ (the function space of "Vanishing Mean Oscillation" (see Definition 2.3 below)).

• (H3) the coefficients $b_i(z)$ and $c(z)$ are measurable functions in Ω,

$$b_i(z)\in\begin{cases} L^{Q+2}, & p+\lambda\le Q+2,\\ L^{p,\lambda}, & p+\lambda>Q+2,\end{cases} \qquad c(z)\in\begin{cases} L^{\frac{Q+2}{2}}, & 2p+\lambda\le Q+2,\\ L^{p,\lambda}, & 2p+\lambda>Q+2.\end{cases}$$

Let

$$\mathcal{H}=\sum_{i,j=1}^{q} a_{ij}(z)X_iX_j-\partial_t. \tag{1.3}$$