JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **22**, No. 4, pp. 334-351

doi: 10.4208/jpde.v22.n4.3
November 2009

The Homotopy Perturbation Method and the Adomian Decomposition Method for the Nonlinear Coupled Equations

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Received 12 November 2008; Accepted 12 August 2009

Abstract. In this paper, we have used the homotopy perturbation and the Adomian decomposition methods to study the nonlinear coupled Kortewge-de Vries and shallow water equations. The main objective of this paper is to propose alternative methods of solutions, which do not require small parameters and avoid linearization and physical unrealistic assumptions. The proposed methods give more general exact solutions without much extra effort and the results reveal that the homotopy perturbation and the Adomian decomposition methods are very effective, convenient and quite accurate to the systems of coupled nonlinear equations.

AMS Subject Classifications: 35K99, 35P05, 35P99

Chinese Library Classifications: O177.7, O177.9

Key Words: Homotopy perturbation method; Adomian decomposition method; coupled Korteweg-de Vries equations; shallow water equations.

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1 Introduction

The nonlinear coupled partial differential equations have many wide array of applications of many fields, which described the motion of the isolated waves, localized in a small part of space, in many fields such as physics, mechanics, biology, hydrodynamics, plasma physics, etc.. To further explain some physical phenomena, searching for exact solutions of nonlinear partial differential equations is very important. Up to now, many researches in mathematical physics have paid attention to these topics, and a lot of

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powerful methods have been presented such as Bäcklund and Darboux transformation [1–3], inverse scattering method [4], complex hyperbolic function method [5], the $(\frac{G'}{G})$-expansion method [6]. The variational iteration method [7–10], Adomian decomposition method [11–21], modified variational iteration method [22] and homotopy perturbation method [18, 21–27] are used to obtain the numerical solutions for the nonlinear partial differential equations. The homotopy perturbation method was first proposed by He in 1999 for solving differential and integral equations, linear and nonlinear has been the subject of extensive analytical and numerical studies. This method has a significant advantage in that it provides an approximate solution to a wide range of nonlinear problems in applied science. The method is a coupling of the traditional perturbation method and homotopy in topology. In topology, a homotopy is constructed with an embedding parameter. Over the last twenty years, the Adomian decomposition method has been applied to obtain formal solutions to a wide class of both deterministic and stochastic differential equations. In resent years, the decomposition method has emerged as an alternative method for solving a wide range of problems whose mathematical models involve algebraic, differential, integral, integro-differential, higher-order ordinary differential equations, partial differential equations and systems [11–20, 28–29]. Our main interest of the present work is being in implementing two reliable and powerful methods namely, the homotopy perturbation and the Adomian decomposition methods to stress the power of these methods to the solutions of the KdV equations [30]

$$u_t - au_{xxx} - 6auu_x - 2bvv_x = 0, \tag{1.1a}$$

$$v_t + v_{xxx} + 3uv_x = 0, \tag{1.1b}$$

and the shallow water equations [30]

$$u_t + uu_x + v_x + \beta u_{xx} = 0, \tag{1.2a}$$

$$v_t + vu_x + uv_x - \beta v_{xx} + \alpha u_{xxx} = 0, \tag{1.2b}$$

where a,b,α,β are real constants. The coupled Kortewege-de Vries equations (1.1) are derived by Hirota and Satsuma to model the interaction of two long water waves with different dispersion relation [31] and considered as the generalized of the KdV equation (see [32] and the references therein). The model system (1.2) is initially derived to describe the dispersive long wave in shallow water [33], where $u(x,t)$ is the field of horizontal velocity, $v(x,t)$ is the height deviating from equilibrium position of liquid and α,β represent different diffusion powers.

2 Homotopy perturbation method

To illustrate the homotopy perturbation method [5, 18, 23–27, 34], we consider a general equation of the type,

$$L(u) = 0, \tag{2.1}$$