J. Partial Diff. Eqs. **21**(2008), 335–346

Vol.21 No.4

THE GENERALIZED RIEMANN PROBLEM FOR A SCALAR COMBUSTION MODEL—THE PERTURBATION ON INITIAL BINDING ENERGY*

Pan Lijun
(Department of Mathematics, Shanghai University, Shanghai 200444; Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 211100, China)
(E-mail: 98010149@163.com)
Sheng Wancheng
(Department of Mathematics, Shanghai University, Shanghai 200444, China)
(E-mail: mathwcsheng@shu.edu.cn)

Dedicated to Professor Li Daqian for his 70th Birthday

(Received Oct. 9, 2007)

Abstract In this paper, we study the generalized Riemann problem for a scalar Chapman-Jouguet combustion model in a neighborhood of the origin on upper half of the (x, t) plane. We focus our attention on the perturbation on initial binding energy. Under the entropy conditions, the solutions are obtained constructively. It shows that the perturbed Riemann solutions possess the structural stability except the case that the corresponding Riemann solutions contain CJDT, for which CJDT may transform into SDT after perturbation on initial binding energy in the neighborhood of the origin.

Key Words Scalar Chapman-Jouguet combustion model; binding energy; perturbation; detonation; deflagration.

2000 MR Subject Classification 35L65, 35L67, 76L05, 76N10, 80A25.

Chinese Library Classification O175.27.

1. Introduction

We are concerned with the simplest combustion model–the Chapman-Jouguet (CJ) combustion model with scalar conservation law,

$$\begin{cases} (u+q)_t + f(u)_x = 0, \\ q(x,t) = \begin{cases} 0, & \sup\limits_{0\le s\le t} u(x,s) > u_i, \\ q(x,0), & \sup\limits_{0\le s\le t} u(x,s) \le u_i, \end{cases} \end{cases} \tag{1.1}$$

*L. Supported by NSFC 10671120, and NUAA's Scientific Fund for the Introduction of Qualified Personnel Shanghai Leading Academic Discipline Project: J50101.

where u is a "lumped variable", representing density, velocity and temperature; q denotes the binding energy of the reactive gas; f represents the flux function with $f'' > 0$; x is the Lagrangian coordinates; $u = u_i$ denotes the ignition temperature. The model describes the combustible gas with a infinite reaction rate. Equivalently, a gas particle in (1.1) releases all of its binding energy once it burns.

In [1, 2], Fickett and Majda independently proposed the simplest Zeldovich-Von Neumann-Döring combustion model with scalar conservation laws,

$$\begin{cases} (u+z)_t + f(u)_x = 0, \\ z_t = -k\phi(u)z, \end{cases} \tag{1.2}$$

where z is the percentage of unburned gas and the constant k represents the rate of chemical reaction. $\phi(u)$ is the standard Heaviside function: $\phi(u) = 0$ as $u \leq u_i$, $\phi(u) = 1$ as $u > u_i$. The model describes the combustible gas with a finite reaction rate. Ying and Teng [3] studied the Riemann problem of (1.2) and obtained the existence and uniqueness of its solution. Furthermore, they proved the existence of limit of the solution as k goes to infinity and found the limit function is a solution of the Riemann problem for the corresponding CJ combustion model. In 2007, Pan and Sheng [4,5] discussed the interactions of shock and combustion waves for the scalar ZND combustion model. By analyzing characteristics in the reaction zone, they constructed the solutions to the problem. Moreover, by studying the limits of the solutions as the reaction rate goes to infinity, they found that the limits are the solutions of the corresponding initial value problem for the scalar CJ combustion model. From Ying and Teng's result, with characteristic method, Liu and Zhang [6] considered the Riemann problem of (1.1) directly and proposed a set of entropy conditions, which consist of the pointwise and the global entropy conditions. The unique solution of (1.1) satisfying the pointwise and the global entropy conditions is exactly the limit of the solution of (1.2). The pointwise entropy solutions, the number of which may be four for some initial data, are the limits of solutions of (1.3), which was proved in [7].

$$\begin{cases} (u+z)_t + f(u)_x = 0, \\ z_t = -\dfrac{k}{t}\phi(u)z. \end{cases} \tag{1.3}$$

For reactive gas flow, it's most interesting to study the stability and the asymptotic behavior of combustion waves [8–11]. In this paper, we study the initial value problem of (1.1) with the following initial value

$$(u, q)(x, 0) = (u_0^{\pm}(x), q_0^{\pm}(x)), \quad \pm x > 0. \tag{1.4}$$

Here $u_0^{\pm}(x)$ and $q_0^{\pm}(x)$ are arbitrary smooth functions with the properties

$$\lim_{x \to 0-} u_0^-(x) = u^-, \quad \lim_{x \to 0+} u_0^+(x) = u^+; \tag{1.5}$$