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LIFE-SPAN OF CLASSICAL SOLUTIONS OF INITIAL-BOUNDARY VALUE PROBLEM FOR FIRST ORDER QUASILINEAR HYPERBOLIC SYSTEMS*

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Abstract In this paper, we consider the mixed initial-boundary value problem for quasilinear hyperbolic systems with nonlinear boundary conditions in a half-unbounded domain $\{(t,x)|\ t \geq 0, x \geq 0\}$. Under the assumption that the positive eigenvalues are not all weakly linearly degenerate,we obtain the blow-up phenomenon of the first order derivatives of C^1 solution with small and decaying initial data.We also give precise estimate of the life-span of C^1 solution.

Key Words Quasilinear hyperbolic system; mixed initial-boundary value problem; life-span; weak linear degeneracy.

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1. Introduction and Main Result

Consider the following first order quasilinear hyperbolic system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = 0, \tag{1.1}$$

where $u = (u_1, \cdots, u_n)^T$ is the unknown vector function of (t, x) and $A(u)$ is an $n \times n$ matrix with suitably smooth elements $a_{ij}(u)$ $(i, j = 1, \cdots, n)$.

By the definition of hyperbolicity, for any given u on the domain under consideration, $A(u)$ has n real eigenvalues $\lambda_1(u), \cdots, \lambda_n(u)$ and a complete set of left (resp. right) eigenvectors. For $i = 1, \cdots, n$, let $l_i(u) = (l_{i1}(u), \cdots, l_{in}(u))$ (resp. $r_i(u) = (r_{i1}(u), \cdots, r_{in}(u))^T$) be a left (resp. right) eigenvector corresponding to $\lambda_i(u)$:

$$l_i(u)A(u) = \lambda_i(u)l_i(u) \tag{1.2}$$

and

$$A(u)r_i(u) = \lambda_i(u)r_i(u). \tag{1.3}$$

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We have

$$\det|l_{ij}(u)| \neq 0 \qquad (resp. \quad \det|r_{ij}(u)| \neq 0). \tag{1.4}$$

Without loss of generality, we suppose that on the domain under consideration

$$l_i(u)r_j(u) \equiv \delta_{ij} \quad (i, j = 1, \cdots, n), \tag{1.5}$$

where δ_{ij} stands for the Kronecker's symbol.

We suppose that all $\lambda_i(u)$, $l_{ij}(u)$ and $r_{ij}(u)$ $(i, j = 1, \cdots, n)$ have the same regularity as $a_{ij}(u)$ $(i, j = 1, \cdots, n)$.

For the Cauchy problem of system (1.1) with the initial data

$$t = 0 : u = \phi(x), \tag{1.6}$$

where $\phi(x)$ is a C^1 vector function with bounded C^1 norm, many results have been obtained (see [1-3] and [4]). In particular, by means of the concept of weak linear degeneracy, for small initial data with certain decaying properties, the global existence and the blow-up phenomenon of C^1 solution to Cauchy problem (1.1) and (1.6) have been completely studied (see [5-9] and [10, 11] , also see [12-15]).

For the mixed initial-boundary value problem of system (1.1) with initial data (1.6) and boundary data

$$x = 0 : v_s = f_s(\alpha(t), v_1, \cdots, v_m) + h_s(t) \quad (s = m+1, \cdots, n), \tag{1.7}$$

on the domain

$$D = \{(t, x)|\ t \geq 0, x \geq 0\}, \tag{1.8}$$

in which

$$v_i = l_i(u)u \quad (i = 1, \cdots, n) \tag{1.9}$$

and

$$\alpha(t) = (\alpha_1(t), \cdots, \alpha_k(t)), \tag{1.10}$$

where $\phi(x), \alpha(t)$ and $h_s(t)(s = m+1, \cdots, n)$ is a C^1 function with certain decay, the global existence of C^1 solution has been obtained under the assumption that the positive eigenvalues are weakly linearly degenerate (see [16]).In order to consider the blow-up phenomenon and the life-span of C^1 solution of system (1.1) and (1.6)–(1.7), in this paper we consider the mixed initial-boundary value problem for system (1.1) in the half-unbounded domain above under the assumption that the positive eigenvalues are not all weakly linearly degenerated. In order to consider global classical solutions and the blow-up phenomenon of initial value problem (see [8] and [10, 11]), it is only necessary to estimate C^1 solution along the characteristic starting from x-axis. However, in this paper, we even need estimate C^1 solution along the characteristic starting from y-axis, actually which can be control by boundary value.

We suppose that the eigenvalues satisfy

$$\lambda_1(0), \cdots, \lambda_m(0) < 0 < \lambda_{m+1}(0) < \cdots < \lambda_n(0). \tag{1.11}$$