J. Partial Diff. Eqs. **20**(2007), 220–246

International Academic Publishers Ltd Vol.20 No.3

LOCAL AND GLOBAL EXISTENCE OF SOLUTIONS OF THE GINZBURG-LANDAU TYPE EQUATIONS*

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(Received Mar. 21, 2006; revised Feb. 26, 2007)

Abstract This paper is devoted to studying the initial value problem of the Ginzburg-Landau type equations. We treat the case where the nonlinear interaction function is a general continuous function, not required to satisfy any smoothness conditions. Local and global existence results of solutions of the problem are given. Decay estimates are also shown.

Key Words Ginzburg-Landau type equations; initial value problem; local existence; global existence.

2000 MR Subject Classification 35Q35, 35K55.

Chinese Library Classification O175.29.

1. Introduction

In this paper we study solvability of the following initial value problem:

$$u_t = (a+ib)\triangle u + f(u,\bar{u},\nabla u,\nabla\bar{u}), \quad x\in\mathbb{R}^n,\ t>0, \tag{1.1}$$

$$u(x,0) = u_0(x), \quad x\in\mathbb{R}^n, \tag{1.2}$$

where $\triangle$ denotes the Laplacian in $\mathbb{R}^n$, u is a complex-valued unknown function of variables $(x,t)\in\mathbb{R}^n\times\mathbb{R}^+$, a, b are real constants, $a>0$, $f(u,v,\zeta,\eta)$ is an arbitrary complex-valued continuous nonlinear function of variables $(u,v,\zeta,\eta)\in\mathbb{C}\times\mathbb{C}\times\mathbb{C}^n\times\mathbb{C}^n$, and $\bar{u}$ represents the complex conjugate of u.

The equation (1.1) is a general form of the Ginzburg-Landau type equations, including the classical Ginzburg-Landau equation

$$u_t = (a+ib)\triangle u + c|u|^2u, \tag{1.3}$$

*This work is supported by the China National Science Foundation (No. 10471157).

the derivative Ginzburg-Landau equation

$$u_t = (a + ib)u_{xx} + c|u|^2 u + c_1 |u|^2 u_x + c_2 u^2 \bar{u}_x \tag{1.4}$$

and various generalized forms of these equations [1-11]. Here a, b are as before, c, c_1, c_2 are complex constants. Such equations are used to describe spatial pattern formation and onset of instabilities in nonequilibrium fluid dynamical systems. Besides, the equation (1.1) also includes the viscous Hamilton-Jacobi equation

$$u_t = \triangle u + c_3 |\nabla u|^p, \tag{1.5}$$

where $c_3 \in \mathbb{R}, c \neq 0$ and $p > 1$, cf. [12]. We refer the reader to see [1-4,6,7,9,12-14] for physical background of the equation (1.1).

Early work on solvability of the problem (1.1)–(1.2) can be found in Doering, Gibbon and Levermore [3], Levermore and Oliver [9], Ginibre and Velo [6, 7]. In these literatures the nonlinear interaction function f has the following form:

$$f = g(|u|^2)u,$$

where g is a C^1-class real-valued function, including the special case $g(s) = s^\gamma$, where γ is a positive constant. Duan and Holmes [4], Gao and Duan [5] and Wang, Guo and Zhao [11] considered nonlinear interaction functions f containing the derivative terms ∇u and $\nabla \bar{u}$ in the form

$$f = c|u|^\sigma u + |u|^\delta (\vec{c}_3 \cdot \nabla u) + |u|^{\delta-2} u^2 (\vec{c}_4 \cdot \nabla \bar{u}) \tag{1.6}$$

or its certain special forms, where σ and δ are positive constants satisfying certain restraints, c is a complex constant, and $\vec{c}_3$, $\vec{c}_4$ are constant complex vectors. Wang [10] considered a more general nonlinear interaction function f which is a more complicated combination of terms that appear in (1.6).

From massive amount of literatures on the Ginzburg-Landau type equations that have already appeared, we see that mathematical expressions of the nonlinear interaction function $f(u, \bar{u}, \nabla u, \nabla \bar{u})$ are quite diversified. Apart from those mentioned above we can easily give more examples. For instance, periodically forced oscillatory reaction-diffusion systems near the Hope bifurcation can be modelled by the resonantly forced Ginzburg-Landau type equation with nonlinear terms

$$f = c|u|^2 u + c_1 \bar{u}^{n-1}$$

and

$$f = c|u|^2 u + c_1 |u|^\sigma$$

(cf. [16,17]). The study of bifurcations with continuous spectrum needs to consider a Ginzburg-Landau type equation with the following interaction term:

$$f = P(u, \bar{u}) + c|u|^\sigma u,$$