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RADIAL MINIMIZER OF P-GINZBURG-LANDAU FUNCTIONAL WITH A WEIGHT*

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Abstract The author discusses the asymptotic behavior of the radial minimizer of the p-Ginzburg-Landau functional with a weight in the case $p > n \geq 2$. The location of the zeros and the uniqueness of the radial minimizer are derived. Moreover, the $W^{1,p}$ convergence of the radial minimizer of this functional is proved.

Key Words Radial minimizer; p-Ginzburg-Landau functional with a weight; asymptotic behavior.

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1. Introduction

The following Ginzburg-Landau functional

$$F(u,G) = \frac{1}{2}\int_G |\nabla u|^2 dx + \frac{1}{4\varepsilon^2}\int_G (1-|u|^2)^2 dx$$

is the simplified energy functional appearing in the study of phase transition problems occurring in superconductivity, superfluids and XY-magnetism. Many papers devoted to studying the asymptotic behavior of the minimizers of this functional in the function class

$$H_g^1(G,R^2) = \{v \in H^1(G,R^2); v|_{\partial G} = g\},$$

where $G \subset R^2$ is a bounded domain with smooth boundary, $g : \partial G \to S^1$ is smooth (see [1] and [2]); and there are also many works to discuss the properties of the radial minimizers (see [3] and [4]), i.e. the minimizers of the functional $F(u,B)$ in the class

$$W = \{u(x) = f(r)\frac{x}{|x|} \in H^1(B,R^2); f(1) = 1, r = |x|\},$$

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where $B = \{x \in R^2; |x| < 1\}$. Meanwhile, asymptotic behavior of the minimizer u_ε of the Ginzburg-Landau energy with a weight

$$E_\varepsilon(u, G) = \frac{1}{2}\int_G |\nabla u|^2 dx + \frac{1}{4\varepsilon^2}\int_G (1 - |u|^2)^2 w(x) dx$$

in the class $H^1_g(G, R^2)$, was well-studied by [5] and [6-9], as dealing with the correlative problem 2 in [1,Ch. 11]. It turns out that as $\varepsilon \to 0$, the minimizer u_ε converges locally to a harmonic map with values in S^1. Moreover, the location of the zeros of minimizer is discussed.

Let $n \geq 2, B = \{x \in R^n; |x| < 1\}$. In this paper, we shall research the asymptotic behavior of the radial minimizer u_ε of the p-Ginzburg-Landau-type functional with a weight

$$E_\varepsilon(u, B) = \frac{1}{p}\int_B |\nabla u|^p + \frac{1}{4\varepsilon^p}\int_B (1 - |u|^2)^2 w, \quad (p > n)$$

on the function class

$$W = \{u(x) = f(r)\frac{x}{|x|} \in W^{1,p}(B, R^n); f(1) = 1, r = |x|\}$$

when $\varepsilon \to 0$. Here $w(r) = w(|x|) \in C^1(\overline{B}, R)$, $w(r_0) = 0$ for some $r_0 \in [0, 1]$, and $w(r) > 0$ in $[0, 1] \setminus \{r_0\}$. By the direct method in calculus of variations, we can see that the minimizer u_ε exists, and it is called the radial minimizer.

We will proved the following

Theorem 1.1 *Let u_ε be a radial minimizer of $E_\varepsilon(u, B)$. Given $\gamma > 0$, denote $A^\gamma_{r_0} = \{x \in \overline{B}; ||x| - r_0| < \gamma\}$. Then there exists a constant h independent of $\varepsilon \in (0, 1)$ such that, $Z_\varepsilon = \{x \in B; |u_\varepsilon(x)| < 2/3\} \subset B(0, h\varepsilon) \cup A^\gamma_{r_0}$. Furthermore, the radial minimizer is unique for any given $\varepsilon \in (0, \varepsilon_0)$ as long as ε_0 is sufficiently small.*

Theorem 1.2 *Assume u_ε is a radial minimizer of $E_\varepsilon(u, B)$, and denote $A_{r_0} = \{x \in \overline{B}; |x| = r_0\}$. Then as $\varepsilon \to 0$,*

$$u_\varepsilon \to \frac{x}{|x|}, \quad in \quad W^{1,p}_{loc}(\overline{B} \setminus (\{0\} \cup A_{r_0}), R^n).$$

Some basic properties of minimizers are obtained in Section 2. The proof of Theorem 1.1 is presented in Section 3. Based on the uniform estimate, which is established in Section 4, we give the proof of Theorem 1.2.

2. Preliminaries

If denote $V = \{f \in W^{1,p}_{loc}(0, 1]; r^{\frac{n-1}{p}} f_r \in L^p(0, 1), r^{(n-1-p)/p} f \in L^p(0, 1), f(1) = 1\}$, then $V = \{f(r); u(x) = f(r)\frac{x}{|x|} \in W\}$. It is not difficult to see