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ON THE INITIAL VALUE PROBLEM FOR THE BIPOLAR SCHRÖDINGER-POISSON SYSTEM

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Abstract In this paper, we prove the existence and uniqueness of global solutions in $H^s(\mathbb{R}^3)$ ($s \in \mathbb{R}, s \geqslant 0$) for the initial value problem of the bipolar Schrödinger-Poisson systems.

Key Words Schrödinger-Poisson system; Strichartz' estimates; initial value problem; H^s-solution.

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1. Introduction

In the present paper, we study the global existence and uniqueness of solutions for the initial value problem to the (pure state) bipolar Schrödinger-Poisson systems

$$i\partial_t\psi = -\Delta\psi + V\psi, \tag{1.1a}$$

$$i\partial_t\phi = -\Delta\phi - V\phi, \tag{1.1b}$$

$$-\Delta V = |\psi|^2 - |\phi|^2, \tag{1.1c}$$

$$\psi(0,x) = \psi_0,\ \phi(0,x) = \phi_0, \tag{1.1d}$$

where $\psi = \psi(t,x)$ and $\phi = \phi(t,x) : \mathbb{R}^{1+3} \to \mathbb{C}$, Δ is the Laplacian operator on $\mathbb{R}^3$, and the electrostatic potential $V = V(\psi,\phi)$ is a real function. This system appears in

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quantum mechanics, semi-conductor and plasma physics. A large amount of interesting works has been devoted to the study for the Schrödinger-Poisson systems (see [1-4] and references therein). In [3], Castella proved the global existence and uniqueness of solutions in $H^m (m \in \mathbb{Z},\ m \geqslant 0)$ for the mixed-state unipolar Schrödinger-Poisson systems. And in [4], Jüngel and Wang discussed the combined semi-classical and quasineutral limit in the bipolar defocusing nonlinear Schrödinger-Poisson system in the whole space.

First, we introduce some notations. For any $p \in [2, \infty)$, we denote $\dfrac{1}{\gamma(p)} = \dfrac{3}{2}(\dfrac{1}{2} - \dfrac{1}{p})$. $S(t)$ denotes the unitary group generated by $i\Delta$ in $L^2(\mathbb{R}^3)$. For $p \in [1, \infty]$, we denote by p' the conjugate exponent of p, defined by $1/p + 1/p' = 1$. $\bar{z}$ denotes the conjugate of the complex number z. H_p^s or $\dot{H}_p^s$ (resp. $B_{p,2}^s$ or $\dot{B}_{p,2}^s$) denotes the inhomogeneous or homogeneous Sobolev (Besov) space respectively.

Now we state the main result of this paper as follows.

Theorem 1.1 *Let $s \in \mathbb{R},\ s \geqslant 0$. Let $a \in [2, \frac{18}{7}]$. Assume that $\psi_0,\ \phi_0 \in H^s(\mathbb{R}^3)$. Then, there exists a unique solution of the IVP* (1.1) *such that* (ψ, ϕ)

$$\psi, \phi \in \mathcal{C}(\mathbb{R}; H^s(\mathbb{R}^3)) \cap L_{loc}^{\gamma(a)}(\mathbb{R}; B_{a,2}^s(\mathbb{R}^3)). \tag{1.2}$$

Moreover, when s is an integer, the result in (1.2) also holds with the Besov space $B_{a,2}^s$ replaced by H_a^s.

Remark The result that we prove here for the single bipolar Schrödinger-Poisson system can be extended to the mixed-state bipolar Schrödinger-Poisson system within the same framework.

2. Global Existence

By (1.1c), we have the potential

$$V(t,x) = \frac{1}{4\pi} \cdot \frac{1}{r} * (|\psi|^2 - |\phi|^2), \tag{2.1}$$

where $r := |x|$. Now we recall the lemma needed to estimate $V(\psi, \phi)\psi$ and $V(\psi, \phi)\phi$.

Lemma 2.1([5, Lemma 1.1]) *Let $0 \leqslant s < \infty,\ 1 \leqslant r' < \infty$. Assume that $l_k, m_k, p_k, q_k > 0$ satisfy*

$$\frac{1}{r'} = \frac{1}{l_k} + \frac{1}{m_k} = \frac{1}{p_k} + \frac{1}{q_k}, \quad k = 0, 1, ..., [s]. \tag{2.2}$$

Then there exists a constant $C > 0$ dependent only on $r',\ n,\ s$ such that

$$\|uv\|_{\dot{B}_{r',2}^s} \leqslant C \sum_{k=0}^{[s]} (\|u\|_{\dot{H}_{p_k}^k} \|v\|_{\dot{B}_{q_k,2}^{s-k}} + \|u\|_{\dot{B}_{l_k,2}^{s-k}} \|v\|_{\dot{H}_{m_k}^k}), \tag{2.3}$$