J. Partial Diff. Eqs. **15**(2002), 81–95
©International Academic Publishers Vol.15 No.4

GLOBAL EXISTENCE AND ASYMPTOTIC BEHAVIOR OF THE SOLUTION TO 1-D ENERGY TRANSPORT MODEL FOR SEMICONDUCTORS

Li Yong and Chen Li
(Academy of Mathematics and System Sciences, CAS,
Beijing 100080, China)
(E-mail: yli@mail.amss.ac.cn dongchli@mx.amss.ac.cn)
(Received Jul. 19, 2002)

Abstract In this paper, we study the asymptotic behavior of global smooth solution to the initial boundary problem for the 1-D energy transport model in semiconductor science. We prove that the smooth solution of the problem converges to a stationary solution exponentially fast as $t \to \infty$ when the initial data is a small perturbation of the stationary solution.

Key Words energy transport model; energy estimate; global existence; asymptotic behavior.

2000 MR Subject Classification 35M10, 35K55, 35K65, 78A35.

Chinese Library Classification O175.29.

1. Introduction

The energy transport(ET) model in semiconductor science, which combines the conservation of mass with balance of energy, can be directly derived from the Boltzmann equation in the diffusion limit [1–5], or obtained formally from the hydrodynamic equations by neglecting some terms [5]. The common form for energy transport model is

$$\begin{aligned}
&\frac{\partial}{\partial t}\rho(\mu,T)+\mathrm{div}J_1=0,\\
&\frac{\partial}{\partial t}U(\mu,T)+\mathrm{div}J_2=\nabla V\cdot J_1+W(\mu,T),\quad \text{in } \Omega,\\
&\lambda^2\triangle V=\rho-C(x),
\end{aligned}\tag{1.1}$$

with

$$\begin{aligned}
J_1&=-L_{11}\Big(\nabla\Big(\frac{\mu}{T}\Big)-\frac{\nabla V}{T}\Big)-L_{12}\nabla\Big(-\frac{1}{T}\Big),\\
J_2&=-L_{21}\Big(\nabla\Big(\frac{\mu}{T}\Big)-\frac{\nabla V}{T}\Big)-L_{22}\nabla\Big(-\frac{1}{T}\Big),
\end{aligned}\tag{1.2}$$

where the unknowns μ, T are chemical potential of the electrons and the electron temperature respectively, V is the electrostatic potential, $\rho(\mu,T)$ the electron density, $U(\mu,T)$ the density of the internal energy, $W(\mu,T)$ the energy relaxation term satisfying $W(\mu,T)(T-T_0)\le 0$ where the positive constant T_0 is the lattice temperature, J_1 the carrier flux density, J_2 the energy flux density, or heat flux, L the diffusion matrices, λ the scaled Debye length, and $C(x)$ the doping profile which represents the background of device. The expression of ρ, U, L and W can be different in real applications.

In a parabolic band structure, the relations for $\rho(\mu,T)$ and $U(\mu,T)$ approximated by Boltzmann statistics are given as

$$\rho(\mu,T)=T^{\frac{3}{2}}\exp\{\frac{\mu}{T}\},\quad U(\mu,T)=\frac{3}{2}\rho T, \tag{1.3}$$

and in the model discussed in [4, 6], one of the most commonly used models

$$L=\mu_0\rho\begin{pmatrix}1 & \frac{3}{2}T\\ \frac{3}{2}T & \frac{15}{4}T^2\end{pmatrix}, \tag{1.4}$$

$$W(\mu,T)=\frac{3}{2}\rho\frac{T_0-T}{\tau_0}, \tag{1.5}$$

where μ_0 is the mobility constant, τ_0 is the relaxation time.

Several authors have studied stationary energy transport models recently, [7–10], and obtained useful results. For the transient case, P. Degond, S. Génieys and A. Jüngel [11] have obtained, for the first time, a result on the weak solution and its large time behavior of a more general parabolic systems by semidiscretization of time and by using entropy function under the physically motivated mixed Dirichlet Neumann boundary condition and initial condition. Furthermore, A. Jüngel has established the regularity and uniqueness when the coefficient matrix $L=L(x)$ in [12]. But unfortunately, in [11] and [12] it is required that L is uniformly positive definite, while the more interesting situation in physical background is that the coefficient matrix is only positive definite. L. Chen and L. Hsiao [13] have directly studied the existence and uniqueness of $(W_p^{2,1}(Q_\tau))^2\times L_q(0,\tau;W_q^2(\Omega))$ solution of (1.1)-(1.5), where L in (1.4) is not uniformly positive definite a priorly.

There are other models in the simulation of semiconductor devices, such as hydrodynamic(HD) and drift diffusion(DD) models. One can find the discussion on the relations of them in [5]. In [14], L. Hsiao and T. Yang have investigated the relation of HD and DD models by comparing the large time behavior of these two models. Since ET and DD models can be obtained from HD model by different scaling, one can expect that the solution of ET model also has the similar large time behavior to HD model.

The main purpose of this paper is to study the global existence and the large time behavior of solutions to (1.1)-(1.5) in one space dimension when the initial datum is around a stationary solution to the corresponding linear DD model. This gives, in certain sense, a description on the relations of these models.