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THE GLOBAL SOLUTION FOR LANDAU-LIFSHITZ-MAXWELL EQUATIONS

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Abstract In this paper, the global existence of a unique smooth solution for the Landau-Lifshitz-Maxwell equations of the ferromagnetic spin chain in $n(1 \leq n \leq 2)$ dimensions is established by using a coupled priori estimates in Sobolev spaces.
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1. Introduction

In 1935, Landau-Lifshitz [1] proposed the following evolution system

$$z_t = \lambda_1 z \times (\Delta z + H) - \lambda_2 z \times (z \times (\Delta z + H)) \tag{1.1}$$

$$\nabla \times H = \frac{\partial E}{\partial t} + \sigma E \tag{1.2}$$

$$\nabla \times E = -\frac{\partial H}{\partial t} - \beta \frac{\partial z}{\partial t} \tag{1.3}$$

$$\nabla \cdot H + \beta \nabla \cdot z = 0, \quad \nabla E = 0 \tag{1.4}$$

where $\lambda_1, \lambda_2, \sigma, \beta$ are constants, $\lambda_2 \geq 0, \sigma \geq 0, z(x,t) = (z_1(x,t), z_2(x,t), z_3(x,t))$ denotes the microscopic magnetization field, $H = (H_1(x,t), H_2(x,t), H_3(x,t))$ the magnetic field, $E(x,t) = (E_1(x,t), E_2(x,t), E_3(x,t))$ the electric field, $H^e = \Delta z + H$ the effective magnetic field, $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}, \nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \cdots, \frac{\partial}{\partial x_n}\right)$, "$\times$" the cross product of the vector in R^3.

If $H = 0, E = 0$, we obtain the Landau-Lifshitz system with Gilbert term

$$z_t = \lambda_1 z \times \Delta z - \lambda_2 z \times (z \times \Delta z) \tag{1.5}$$

where $\lambda_2 > 0$ is a Gilbert damping coefficient. In [2–4], the properties of the solution for the system of the equation (1.5) and the closed new links between the solution and the harmonic map on the compact Riemann manifold have been studied extensively. When $\lambda_2 = 0$, the system of the equation (1.5) becomes

$$z_t = \lambda_1 z \times \Delta z \tag{1.6}$$

In the case of $n = 1$, it is an integral system, and has N soliton solutions. In [5–13], one has studied in detail for (1.6) the interaction between solitons, infinite conservative laws, inverse scattering method, and the relation with the nonlinear Schrödinger equations. As pointed out in [14], the system of the equation (1.6) is a strongly coupled degenerate quasilinear parabolic system. In [14–22], we have investigated extensively the classic and generalized solutions to the initial value problem and other kinds of boundary value problem for the system of the equation (1.6), and some properties of the solutions, and further obtained the global generalized solutions for $n \geq 2$.

In this paper we shall discuss the existence of the global solution to the periodic initial value problem of the system (1.1)–(1.4):

$$\begin{aligned} &z(x + 2De_i, t) = z(x,t), \quad H(x + 2De_i, t) = H(x,t) \\ &E(x + 2De_i, t) = E(x,t), \quad t \geq 0 \end{aligned} \tag{1.7}$$

$$\begin{aligned} &z(x_1, \cdots, x_n, 0) = z_0(x), \quad H(x_1, \cdots, x_n, 0) = H_0(x) \\ &E(x_1, \cdots, x_n, 0) = E_0(x), \quad x \in \Omega \end{aligned} \tag{1.8}$$

where $x + 2De_i = (x_1, \cdots, x_{i-1}, x_i + 2D, x_{i+1}, \cdots, x_n)(i = 1, 2, \cdots, n), D > 0, \Omega \subset R^2$ is n-dimensional cube with width $2D$; or the initial value problem:

$$z(x,0) = z_0(x), \quad H(x,0) = H_0(x), \quad E(x,0) = E_0(x), \quad x \in R^n \tag{1.9}$$

In Section 2 we first derive the *a priori* estimates. In Section 3 we prove the existence and uniqueness of the global classical solution for the periodic initial value problem of the system (1.1)–(1.4), (1.7), (1.8) and the initial value problem (1.1)–(1.4), (1.9).

2. A *Priori* Estimates

Lemma 1 *Let* $|z_0(x)| = 1$. *Then for the smooth solution of the periodic initial value problem of the system* (1.1)–(1.4), (1.7), (1.8)

$$|z(x,t)| = 1, \quad x \in \Omega, \quad t \geq 0 \tag{2.1}$$

Proof Making the scalar product of z with (1.1), we get

$$z \cdot z_t = 0, \quad |z(x,t)|_t^2 = 0$$

Then

$$|z(x,t)| = |z_0(x)| = 1$$