J.Partial Diff. Eqs. **13**(2000), 97–110
©International Academic Publishers Vol.13, No.2

GLOBAL ATTRACTOR FOR WEAKLY DAMPED NONLINEAR SCHRÖDINGER-BOUSSINESQ EQUATIONS IN AN UNBOUNDED DOMAIN*

Gao Ping
(Department of Mathematics, Qujing Educational Institute, Qujing 655000, China)
Dai Zhengde
(Department of Mathematics, Yunnan University, Kunming 650091, China)
(Received Jan. 12, 1998)

Abstract In this paper the authors consider the Cauchy problem of nonlinear Schrödinger-Boussinesq equations in **R** and prove the existence of the maximal attractor.

Key Words Nonlinear Schrödinger-Boussinesq; bounded absorbing set; decomposition of operator; maximal attractor.
1991 MR Subject Classification 35Q55, 35Q53, 58F39.
Chinese Library Classification O175.2, O175.29.

1. Introduction

In this paper, We consider the following nonlinear Schrödinger-Boussinesq equations

$$i\varepsilon_t + \Delta\varepsilon - n\varepsilon - \beta|\varepsilon|^2\varepsilon + i\gamma\varepsilon = g(x) \tag{1.1}$$

$$n_t = \Delta\varphi \tag{1.2}$$

$$\varphi_t = n + \mu n_t - \lambda\Delta n - \alpha\varphi + |\varepsilon|^2 \tag{1.3}$$

$$\varepsilon(0,x) = \varepsilon_0, \quad n(0,x) = n_0, \quad \varphi(0,x) = \varphi_0, \quad x \in R, \quad t \in R^+ \tag{1.4}$$

where $\alpha, \beta, \gamma, \mu, \lambda > 0$ are constants, $\varepsilon(x,t)$ is comlex, $n(x,t)$, $\varphi(x,t)$ are real-valued.

This model arose from the laser and plasma physics under interaction of nonlinear complex Schrödinger field and real Boussinesq field. In [1] the authors considered the one-dimensional dissipative SBq which involves a three order nonlinear term in the Schrödinger equation and a strong dissipativity in the Boussinesq equation. They proved the existence of the global attractor and the finiteness of the attractor in the weak topology sense. In this paper we are going to prove that (1.1)–(1.4) possesses a maximal attrator in $H^2 \times H^2 \times H^2(R)$ which attracts bounded sets of $H^3 \times H^3 \times H^3(R)$ in the topology of the $H^2 \times H^2 \times H^2(R)$.

*The project supported by National Natural Science Foundation of China (No.19861004).

We note that, in the unbounded domain case of our present paper, the imbedding of $H^s(R)$ into $H^{s'}(R)$ $(s > s')$ is not compact. To overcome this difficulty we adapt the methods in [2] and in [3] and utilize the Kuratowskii α-measure of noncompactness to show the asymptotic smoothness of the semigroup $S(t)$. Then we can apply the theory of [4] to prove the existence of the maximal attractor.

We introduce the following standard notations., we denote the spaces of complex valued functions and real valued functions by the same symbols. For $s \geq 0, 1 \leq p < \infty$, $H^{s,p}(R)$ is the usual Sobolev spaces of orders s. $H^s(R) = H^{s,2}(R)$. $(\cdot, \cdot)$ denotes the inner product in $L^2(R)$, we denote by $\|\cdot\|_p$ the norm of $L^p(R)$ and by $\|\cdot\|_{H^s}$ the norm of $H^s(R)$. Especially $\|\cdot\| = \|\cdot\|_2$, we set $V = H^1 \times H^1 \times H^1(R)$.

$$X = H^2 \times H^2 \times H^2(R), \quad Y = H^3 \times H^3 \times H^3(R)$$

Then $Y \hookrightarrow X \hookrightarrow V$ with continuous imbedding. For any Banach space E, $C_b(I; E)$ denotes the space of continuous and bounded functions on an interval $I \subset R$ with values in E. C is a generic constant and may assume various values from line to line.

2. Global Solutions and Bounded Absorbing Sets

We introduce a transformation $m = n_t + \rho n$, where ρ is a positive constant. Then the problem (1.1)–(1.4) is equivalent to the following:

$$i\varepsilon_t + \Delta\varepsilon - n\varepsilon - \beta|\varepsilon|^2\varepsilon + i\gamma\varepsilon = g(x) \tag{2.1}$$

$$n_t = \Delta\varphi \tag{2.2}$$

$$\varphi_t = n + \mu n_t - \lambda\Delta n - \alpha\varphi + |\varepsilon|^2 \tag{2.3}$$

$$(\varepsilon, n, \varphi)(0, x) = (\varepsilon_0, n_0, \varphi_0)(x), \quad x \in R \tag{2.4}$$

$$m = n_t + \rho n \tag{2.5}$$

In this section we establish time-uniform *a priori* estimates of solutions $(\varepsilon, n, \varphi)$ in V, nest in X and then in Y, which guarantee the existence of the global solutions and bounded absorbing sets.

Lemma 2.1 *Let $g(x) \in H^2(R)$. Then*
1° for any $(\varepsilon_0, n_0, \varphi_0) \in V$, the solution of (2.1)–(2.5) belongs to $L^\infty(R^+; V)$;
2° for any $(\varepsilon_0, n_0, \varphi_0) \in X$, the solution of (2.1)–(2.4) belongs to $L^\infty(R^+, X)$;
3° for any $(\varepsilon_0, n_0, \varphi_0) \in Y$, the solution of (2.1)–(2.4) belongs to $L^\infty(R^+, Y)$.

Proof Taking the analogous procedure as in [1], we can obtain the following

(1)
$$\frac{d}{dt}\|\varepsilon\|^2 + 2\gamma\|\varepsilon\|^2 = 2\mathrm{Im}\int g\bar{\varepsilon}dx \tag{2.6}$$

(2) Let

$$H_1(t) = \|\nabla\varepsilon\|^2 + \int n|\varepsilon|^2dx + \beta/2\int|\varepsilon|^4dx + 2\mathrm{Re}\,(g, \varepsilon) + 1/2\|\Delta^{-1/2}m\|^2$$