J.Partial Diff. Eqs. **13**(2000), 111–122
©International Academic Publishers Vol.13, No.2

LIMIT BEHAVIOUR OF SOLUTIONS TO A CLASS OF EQUIVALUED SURFACE BOUNDARY VALUE PROBLEMS FOR PARABOLIC EQUATIONS*

Li Fengquan
(Department of Mathematics, Qufu Normal University, Qufu 273165, Shandong, China)
(Received Jul. 10, 1998; revised Oct. 14, 1998)

Abstract In this paper, we discuss the limit behaviour of solutions for a class of equivalued surface boundary value problems for parabolic equations. When the equivalued surface boundary $\tilde{\Gamma}_1^\varepsilon$ shrinks to a fixed point on boundary Γ_1, only homogeneous Neumann boundary conditions or Neumann boundary conditions with Dirac function appear on Γ_1.
Key Words Parabolic equations; equivalued surface; limit behaviour; Dirac function.
1991 MR Subject Classification 35K10, 35K20.
Chinese Library Classification O175.2, O175.26.

1. Introduction and the Main Results

In many practical applications, especially by resistivity well-logging in petroleum exploitation, the equivalued surface boundary value problem is formulated (cf. [1–3]). From the formulation of the equivalued surface boundary value condition and its physical sense, it is corresponding to a source on the equivalued surface boundary. In the two-dimensional case, this is a line source; in the three- or more-dimensional case, this is a surface or hypersurface source. When the equivalued surface boundary shrinks to a point, this type of source is changed into a point source. Case 1: when the equivalued surface boundary is inside of the domain, the limit behaviour of solutions had been discussed in [4–6]; Case 2: when the equivalued surface boundary shrinks to a fixed point on the boundary, the limit behaviour of solutions to elliptic equations had been discussed in [3, 7–9]. This paper discusses the limit behaviour of solutions to parabolic equations in Case 2.

Ω is a bounded open set in $\boldsymbol{R}^n$ $(n = 2, 3)$ with smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$ (Γ_1 being the outer boundary and $\Gamma_0 \neq \emptyset$ being the interior boundary with $\Gamma_1 \cap \Gamma_0 = \emptyset$). For any fixed $\varepsilon > 0$, we assume that Γ_1 is partitioned into two subsets $\tilde{\Gamma}_1^\varepsilon$ and Γ_1^ε, furthermore $\tilde{\Gamma}_1^\varepsilon$ containing the origin (see Fig.1). T is a fixed positive constant, $Q = \Omega \times (0, T)$, $\Sigma_0 = \Gamma_0 \times (0, T)$, $\Sigma_1 = \Gamma_1 \times (0, T)$, $\Sigma_1^\varepsilon = \Gamma_1^\varepsilon \times (0, T)$, $\tilde{\Sigma}_1^\varepsilon = \tilde{\Gamma}_1^\varepsilon \times (0, T)$.

* The work supported by NSF of Shandong province.

We consider the following initial-boundary value problem:

$$(I_\varepsilon)\begin{cases} u_\varepsilon' + Lu_\varepsilon = 0 & \text{in } Q \\ \dfrac{\partial u_\varepsilon}{\partial n_L} = 0 & \text{on } \Sigma_1^\varepsilon \\ u_\varepsilon = C_\varepsilon(t)(\text{unknown function of } t) & \text{on } \tilde{\Sigma}_1^\varepsilon \\ \displaystyle\int_{\tilde{\Gamma}_1^\varepsilon} \frac{\partial u_\varepsilon}{\partial n_L} ds = A_\varepsilon(t), & \text{a.e. } t \in (0,T) \\ u_\varepsilon = 0 & \text{on } \Sigma_0 \\ u_\varepsilon(x,0) = 0 & \text{on } \Omega \end{cases}$$

where A_ε is a known function in $L^2(0,T)$, $u_\varepsilon' = \dfrac{\partial u_\varepsilon}{\partial t}$, and

$$Lu_\varepsilon = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\Big(a_{ij}\frac{\partial u_\varepsilon}{\partial x_j}\Big) \tag{1.1}$$

$$\frac{\partial u_\varepsilon}{\partial n_L} = \sum_{i,j=1}^{n} a_{ij}\frac{\partial u_\varepsilon}{\partial x_j} n_i \tag{1.2}$$

denotes the co-normal derivative and $n = \{n_1, n_2, \cdots, n_n\}$ denotes the unit outward normal vector on Γ_1.

We make the following assumptions:

(H_1) $a_{ij} \in W^{1,\infty}(\Omega)$, $(i,j = 1,\cdots,n)$, and satisfy

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \lambda_0|\xi|^2, \quad \forall \xi \in R^n, \text{ a.e. } x \in \Omega \tag{1.3}$$

for a fixed positive constant λ_0.

(H_2) Suppose that for any small $\varepsilon > 0$, Γ_1^ε is connected, and if $0 < \varepsilon_1 < \varepsilon_2$, then $\bar{\Gamma}_1^{\varepsilon_1} \subset \bar{\Gamma}_1^{\varepsilon_2}$ and as ε goes to zero, $\bar{\Gamma}_1^\varepsilon \to \{0\}$.

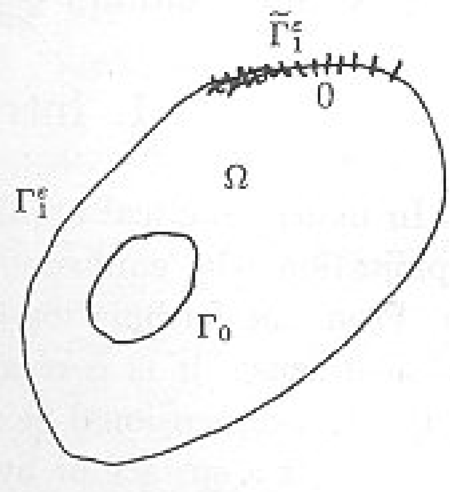

Fig.1

Defining the solution u_ε to the problem (I_ε) by the transposition method and using Green's formula

$$\int_Q u_\varepsilon \psi_\varepsilon dx dt = \int_0^T d_\varepsilon(t) A_\varepsilon(t) dt, \quad \forall \psi_\varepsilon \in L^2(Q) \tag{1.4}$$

where v_ε is the solution to the adjoint problem (II_ε) for problem (I_ε),

$$(II_\varepsilon)\begin{cases} -v_\varepsilon' + L^* v_\varepsilon = \psi_\varepsilon & \text{in } Q \\ \dfrac{\partial v_\varepsilon}{\partial n_L} = 0 & \text{on } \Sigma_1^\varepsilon \\ v_\varepsilon = d_\varepsilon(t) \text{ (unknown function of } t) & \text{on } \tilde{\Sigma}_1^\varepsilon \\ \displaystyle\int_{\tilde{\Gamma}_1^\varepsilon} \frac{\partial v_\varepsilon}{\partial n_L} ds = 0, & \text{a.e. } t \in (0,T) \\ v_\varepsilon = 0 & \text{on } \Sigma_0 \\ v_\varepsilon(x,T) = 0 & \text{on } \Omega \end{cases}$$