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ORBITAL STABILITY OF SOLITARY WAVES OF THE NONLINEAR SCHRÖDINGER-KDV EQUATION

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Abstract This paper concerns the orbital stability of solitary waves of the system of KdV equation coupling with nonlinear Schrödinger equation. By applying the abstract results of Grillakis et al. [1–2] and detailed spectral analysis, we obtain the stability of the solitary waves.
Key Words Solitary wave; stability; nonlinear Schrödinger-KdV equation.
Classification 35Q55, 35B35.

1. Introduction

In this paper, we consider the following nonlinear Schrödinger-KdV system

$$\begin{cases} i\varepsilon_t + \varepsilon_{xx} - n\varepsilon = 0 \\ n_t + n_x + \beta(n^2)_x + \alpha n_{xxx} = -(|\varepsilon|^2)_x \end{cases} \quad x \in \mathbf{R} \tag{1.1}$$

with n a real function and ε a complex function. The problem (1.1) arises in laser and plasma physics. The local and global existence of initial value problem for (1.1) was considered in [3].

In this paper, we consider the stability of solitary waves of (1.1). By applying the abstract theory of Grillakis et al. [1–2] and detailed spectral analysis, we obtain the sufficient conditions for the stability of the solitary waves.

For the other types of equations, such as nonlinear Schrödinger equation, KdV equation and BO equation, the orbital stability of solitary waves were considered in [1–2, 4–8].

This paper is organized as follows: in Section 2, we state the results of the existence of solitary waves; in Section 3, we state the assumptions and the stability results; in Section 4, we obtain the sufficient conditions for the stability.

2. The Existence of Solitary Waves

Consider the following nonlinear Schrödinger-KdV system

$$\begin{cases} i\varepsilon_t + \varepsilon_{xx} - n\varepsilon = 0 \\ n_t + n_x + \beta(n^2)_x + \alpha n_{xxx} = -(|\varepsilon|^2)_x \end{cases} \quad x \in \mathbf{R} \tag{2.1}$$

Let

$$\varepsilon(t,x) = e^{-i\omega t} e^{iq(x-vt)} \widehat{\varepsilon}_{\omega,v}(x-vt) \tag{2.2}$$

$$n(t,x) = n_{\omega,v}(x-vt) \tag{2.3}$$

be the solitary waves of (2.1), where ω, q, v are real numbers, $\widehat{\varepsilon}_{\omega,v}$ and $n_{\omega,v}$ are real functions. Put (2.2)–(2.3) into (2.1), we obtain

$$\widehat{\varepsilon}''_{\omega,v} + i(2q-v)\widehat{\varepsilon}'_{\omega,v} + (\omega + qv - q^2 - n_{\omega,v})\widehat{\varepsilon}_{\omega,v} = 0 \tag{2.4}$$

$$-(v-1)n_{\omega,v} + \beta n^2_{\omega,v} + \alpha n''_{\omega,v} + \widehat{\varepsilon}^2_{\omega,v} = 0 \tag{2.5}$$

(2.4) implies

$$2q = v \tag{2.6}$$

Let $\widehat{\varepsilon}_{\omega,v} = c_1 \operatorname{sech} c_2 x$ satisfy (2.4) with constants c_1, c_2 determined later, then we have

$$\widehat{\varepsilon}''_{\omega,v} = (c_2^2 - 2c_2^2 \operatorname{sech}^2 c_2 x)\widehat{\varepsilon}_{\omega,v} = \Big(-\omega - \frac{v^2}{4} + n_{\omega,v}\Big)\widehat{\varepsilon}_{\omega,v} \tag{2.7}$$

Suppose $n_{\omega,v} \to 0$, as $x \to \infty$, by (2.7), we have

$$n_{\omega,v} = -2c_2^2 \operatorname{sech}^2 c_2 x + c_2^2 + \omega + \frac{v^2}{4} = -2c_2^2 \operatorname{sech}^2 c_2 x \tag{2.8}$$

$$c_2^2 = -\omega - \frac{v^2}{4} \tag{2.9}$$

Put (2.8), (2.9) into (2.5), we have

$$2(v-1)c_2^2 \operatorname{sech}^2 c_2 x + 4\beta c_2^4 \operatorname{sech}^4 c_2 x = 4\alpha c_2^4 (2\operatorname{sech}^2 c_2 x - 3\operatorname{sech}^4 c_2 x) - c_1^2 \operatorname{sech}^2 c_2 x \tag{2.10}$$

It follows from (2.6), (2.9) and (2.10) that

$$\begin{cases} \alpha = -\frac{1}{3}\beta, \quad \beta > 0 \\ q = \frac{v}{2} \\ c_2 = \sqrt{-\omega - \frac{v^2}{4}} \\ c_1 = \sqrt{2\Big(-\omega - \frac{v^2}{4}\Big)\Big(1 - v - \frac{4}{3}\beta\Big(-\omega - \frac{v^2}{4}\Big)\Big)}, \quad v < 1 \end{cases}$$