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ASYMPTOTIC BEHAVIOR OF THE SOLUTION TO A MODEL SYSTEM IN LINEARLY VISCOUS MATERIALS*

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Abstract In this paper, we investigate the asymptotic behavior of solution to a model system in linearly viscous materials with temperature-dependent viscosity.

Key Words Asymptotic behavior; smooth solution; model system; temperature-dependent viscosity; linearly viscous materials.

Classification 35Q72, 35B40.

1. Introduction

In this paper we investigate the asymptotic behavior of solution to a system of partial differential equations for a fairly general class of linearly viscous materials. The balance laws of mass, momentum and energy for one-dimensional case (in Lagrangian form) are

$$u_t - v_x = 0 \tag{1.1}$$

$$v_t - \Big(\frac{\mu(\theta,u)}{u}\frac{\partial v}{\partial x}\Big)_x - \frac{\partial p(\theta,u)}{\partial x} = 0 \tag{1.2}$$

$$\theta_t - \Big(\frac{k(\theta,u)}{u}\frac{\partial \theta}{\partial x}\Big)_x - \frac{\mu(\theta,u)}{u}\Big(\frac{\partial v}{\partial x}\Big)^2 - p(\theta,u)\frac{\partial v}{\partial x} = 0 \tag{1.3}$$

where u is the specific volume, v is the velocity, θ is the absolute temperature, p is the pressure, e is the internal energy, μ and k denote the coefficients of viscosity and heat conductivity respectively.

The system is supplemented with the equations of state

$$e = e(\theta, u), \quad p = p(\theta, u) \tag{1.4}$$

where $e(\theta,u)$ and $p(\theta,u)$ are chosen so as to satisfy the second law of thermodynamics.

When the material is an ideal linearly viscous gas with constant viscosity and heat conductivity, e.g.

$$e = C_V\theta, \quad p = R\frac{\theta}{u}, \quad \mu = \text{const} > 0, \quad k = \text{const} > 0 \tag{1.5}$$

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the initial boundary value problems for (1.1)–(1.3) were studied by Kazhikhov [1], Kazhikhov and Shelukhin [2], Nagasawa [3]. Their method depends crucially upon the one-dimensional form of equations and the specific form of the constitutive relations (1.5). However, under very high temperature and density the relations (1.5) become unsuitable since the heat conductivity and viscosity vary greatly with the temperature and density.

The global existence and asymptotic behavior for the system (1.1)–(1.4) have been established only for special forms of function $\mu(\theta, u)$ and $k(\theta, u)$. For solid-like thermoviscoelastic materials, we refer to Dafermos [4], Dafermos and Hsiao [5] for global existence and to Hsiao and Luo [6] for large time behavior of solutions. For the thermoviscoelastic system in shape memory alloys, W. Shen, S. Zheng and P. Zhu [7], or P. Zhu [8] proved the global existence and asymptotic behavior of weak solutions. For the heat conductive real gas the results of [1, 2] were generalized by Kawohl [9].

The main restriction in the above mentioned papers is that μ does not depend on the temperature. This is certainly a restriction which is not physically motivated, because in general the viscosity does vary with temperature. From the mathematical point of view, the dependence of the viscosity on temperature is also very interesting because a more serious nonlinearity is involved. More precisely, one can formally seperate parabolic system for v and θ from the first order differential equation for u. The dependence of μ and k only on u extends the nonlinearity of the whole system (1.1)–(1.4) but it does not change the nonlinearity of the seperated parabolic system or the equation for u. The dependence of μ and k on θ changes the situation very much and the first dependence "worsens" both of the parabolic equations.

It should be observed that Dafermos and Hsiao [10] were first to consider some initial boundary value problems for the equations obtained from the system (1.1)–(1.3) when u=const, $k = p = 0$, but the viscosity varies with the temperature. They have studied the problem provided by adiabatic rectilinear shearing flow of an incompressible viscous fluid between two parallel plates.

In this paper we consider a model system which can be called the generalization of Burger's equation of a viscous compressible fluid (cf. [11, 12]) for the heat conductive case.

$$u_t - v_x = 0 \tag{1.6}$$

$$v_t - \Big(\frac{\mu(\theta)}{u}\frac{\partial v}{\partial x}\Big)_x = 0 \tag{1.7}$$

$$\theta_t - \Big(\frac{k(\theta)}{u}\frac{\partial \theta}{\partial x}\Big)_x - \frac{\mu(\theta)}{u}\Big(\frac{\partial v}{\partial x}\Big)^2 = 0 \tag{1.8}$$

The function $\mu(\theta)$ is typically decreasing in the case of liquids. We consider the function of viscosity with the following properties:

$$\mu(s) \in C^2(R^+), R^+ = \{x \in R; x \geq 0\} \tag{1.9}$$