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GLOBAL WEAK SOLUTIONS OF THE CAUCHY PROBLEM TO A HYDRODYNAMIC MODEL FOR SEMICONDUCTORS

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Abstract In this paper we are concerned with the global existence of weak solutions of the Cauchy problem for a simplified one-dimensional hydrodynamic model for semiconductors. Convergence of approximate solutions derived by the fractional step Lax-Friedrichs scheme is established by using the compensated compactness method.

Key Words Hydrodynamic model; Lax-Friedrichs scheme; compensated compactness.

Classification 35L70, 35Q60.

1. Introduction

We consider the following hydrodynamic model for semiconductor devices

$$\begin{cases} n_t + (nu)_x = 0 \\ (nu)_t + (nu^2 + p(n))_x = nE - \dfrac{nu}{\tau} \\ E_x = n - b(x) \end{cases} \tag{1.1}$$

in the region $\Pi_T = R \times (0, T)$, for some fixed $T > 0$. Here $n \geq 0$ denotes the electron density, u the (average) particle velocity and E the (negative) electric field, which is generated by the Coulomb force of the particles. The function $b = b(x) \geq 0$ stands for the density of fixed, positively charged background icons, $p = p(n)$ is the pressure-density relation, particularly we shall use here $p(n) = \dfrac{n^\gamma}{\gamma}$ $(\gamma \geq 2)$.

For simplicity we assume $\tau = 1$. Also $J = nu$ will denote the electron current density, which is the conservative state variable.

The hydrodynamic model (1.1) plays an important role in simulating the behavior of the charge carrier in submicron semiconductor devices since it exhibits velocity overshoot and ballistic effects which are not accounted for in the classical drift-diffusion model [1, 2]. The hydrodynamic model consists of a set of nonlinear Euler-Poisson

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equations for particle density, current density, energy density and the electrostatic potential. In this article we investigate (1.1) with the initial value condition

$$(n(x,0),u(x,0)) = (n_0(x),u_0(x)) \tag{1.2}$$

in $\boldsymbol{R}$, where $n_0(x)$ and $u_0(x)$ have compact supports.

After introducing the current density $J = nu$, we can rewrite (1.1)–(1.2) as follows

$$\begin{cases} n_t + J_x = 0 \\ J_t + \left(\dfrac{J^2}{n} + p(n)\right)_x = nE - J \\ E_x = n - b(x) \end{cases} \tag{1.3}$$

in Π_T and

$$(n(x,0),J(x,0)) = (n_0(x),J_0(x)) \tag{1.4}$$

in $\boldsymbol{R}$.

In Degond-Markowich [3], the existence for the one-dimensional steady-state equations was obtained in the subsonic case, and in Gamba [4], a viscosity method was used to study the boundary layers that appear when the viscosity coefficient vanishes. In Zhang [5] and Marcati-Natalini [6], they investigated the initial-boundary value problem and the Cauchy problem of (1.1), respectively. Both of them obtained the global weak solution of the model under consideration for the pressure-density $p(n) = n^\gamma/\gamma$ $(1 < \gamma < 5/3)$. Theorem 5.1 of [6], which holds only for the case that the classical mechanical entropy is strictly convex, is crucial to the whole proof of [6]. But this result will not be true for the case $\gamma \geq 2$, in which the strict convexity of the classical mechanical entropy does not work. We have established a similar crucial result by using a different method from [6] in order to overcome the above difficulty. Then, we can extend the result of [6] to the case $\gamma \geq 2$.

We want to point out that the theory of nonhomogeneous hyperbolic systems of quasi-linear equations is not yet fully researched, some interesting results related to various kinds of local source terms have been made in [7–10].

DiPerna [11, 12] and Chen [13] made a detailed analysis and established some framework theorems for hyperbolic conservation laws by using the theory of compensated compactness. DiPerna [11] obtained a compactness framework for the viscosity method applied to the isentropic system of gas dynamics for $\gamma = 1 + \dfrac{2}{2n+1}$ (integers $n \geq 2$). And Chen [13] generalized this compactness framework in the case $1 < \gamma \leq 5/3$. P.L. Lions et al. [14, 15] extended successfully this compactness framework to the case $\gamma > 1$ through the theory of kinetic formulation of hyperbolic conservation laws. The crucial idea of all the results mentioned above is to show that a family of Young measures corresponding to uniformly bounded approximate solutions reduces to a family of Dirac measures. One achieves this aim by showing that a family of entropy dissipation