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CONVERGENCE OF ITERATIVE DIFFERENCE METHOD WITH NONUNIFORM MESHES FOR QUASILINEAR PARABOLIC SYSTEMS*

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Abstract In this paper, we study the general difference schemes with nonuniform meshes for the following problem:

$$u_t = A(x,t,u,u_x)u_{xx} + f(x,t,u,u_x), \quad 0 < x < l, \quad 0 < t \le T \tag{1}$$

$$u(0,t) = u(l,t) = 0, \quad 0 < t \le T \tag{2}$$

$$u(x,0) = \varphi(x), \quad 0 \le x \le l \tag{3}$$

where u, φ, and f are m-dimensional vector valued functions, $u_t = \dfrac{\partial u}{\partial t}$, $u_x = \dfrac{\partial u}{\partial x}$, $u_{xx} = \dfrac{\partial^2 u}{\partial x^2}$. In the practical computation, we usually use the method of iteration to calculate the approximate solutions for the nonlinear difference schemes. Here the estimates of the iterative sequence constructed from the iterative difference schemes for the problem (1)–(3) is proved. Moreover, when the coefficient matrix $A = A(x,t,u)$ is independent of u_x, the convergence of the approximate difference solution for the iterative difference schemes to the unique solution of the problem (1)–(3) is proved without imposing the assumption of heuristic character concerning the existence of the unique smooth solution for the original problem (1)–(3).

Key Words Difference scheme; nonlinear parabolic systems; iteration; convergence.

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1. Introduction

1. The difference solutions of the nonlinear difference schemes are usually computed by the method of iteration, and on each layer the iteration procedure must be stoped at

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the finite step of iteration. Moreover the approximate value of the difference solution for the difference scheme on each layer is taken to be the initial value of the iteration process on next layer. Such kinds of difference schemes are called by the iterative difference schemes. For the problem (1)–(3) we have proved in [1] the convergence of the iterative difference schemes with nonuniform meshes under the assumption of the existence of the unique smooth solution for the original problem.

2. Suppose that the following conditions are fulfilled.

(I) For the coefficient matrix $A(x,t,u,p)$, there is a positive constant $\sigma_0 > 0$, such that for any $\xi \in \mathbf{R}^m$ and $(x,t) \in \overline{Q}_T$, $u, p \in \mathbf{R}^m$

$$(\xi, A(x,t,u,p)\xi) \geq \sigma_0|\xi|^2 \tag{4}$$

is valid.

(II) The m-dimensional vector function $f(x,t,u,p)$ is continuous with respect to the variables $(x,t) \in \overline{Q}_T$ and the vector variables $u, p \in \mathbf{R}^m$ and there exists a constant C such that

$$|f(x,t,u,p)| \leq C(|p| + |u| + 1)$$

for $(x,t,u,p) \in \overline{Q}_T \times \mathbf{R}^{2m}$.

(III) The initial vector function $\varphi(x) \in C^{(1)}(0,l)$ and satisfies $\varphi(0) = \varphi(l) = 0$. Here for the sake of simplicity we assume the homogeneous boundary condition (2).

(IV) The $m \times m$ matrix function $A(x,t,u,p)$ is continuous with respect to the variables $(x,t) \in \overline{Q}_T$ and the vector variable $u, p \in \mathbf{R}^m$.

2. Iterative Difference Schemes

3. Here and below, we adopt the same notations and symbols as those in [2] and [1].

Define

$$\overset{(s)}{v}{}_j^0 = \varphi_j = \varphi(x_j), \quad s = 0, 1, \cdots, s_{-\frac{1}{2}} \tag{5}$$

where $s_{-\frac{1}{2}}$ is a positive integer. For each n $(0 \leq n \leq N-1)$ define a finite sequence of discrete vector functions $\{\overset{(s)}{v}{}_j^{n+1} \mid j = 0, 1, \cdots, J\}$ $(s = 0, 1, \cdots, s_{n+\frac{1}{2}})$ as follows: $\overset{(0)}{v}{}_j^{n+1} = \overset{(s_{n-\frac{1}{2}})}{v}{}_j^{n}$, $\{\overset{(s+1)}{v}{}_j^{n+1} \mid j = 0, 1, \cdots, J\}$ is the solution of the following difference scheme

$$\frac{\overset{(s+1)}{v}{}_j^{n+1} - \overset{(s_{n-\frac{1}{2}})}{v}{}_j^{n}}{\tau^{n+\frac{1}{2}}} = \overset{(s)}{A}{}_j^{n+\alpha} \delta^2 \overset{(s+1)}{v}{}_j^{n+\alpha} + \overset{(s)}{f}{}_j^{n+\alpha} \tag{6}$$