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BUBBLINGS OF A FAMILY OF SINGULAR PSEUDO-HOLOMORPHIC DISKS

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Abstract In this paper we will discuss the bubblings at corners of a family of pseudo-holomorphic disks with their boundaries in a Lagrangian subvariety with corners.

Key Words Pseudo-holomorphic disk; bubbling; moduli space; corner.
Classification 35J65.

1. Introduction

It is well-known that there are several important applications of elliptic differential equations to the topology and geometry of manifolds and that the moduli spaces of the solutions to these differential equations carry some invariants of the concerned manifolds, on which the corresponding differential equations are defined, for example, Yang-Mills equations and Seiberg-Witten equations on 4-manifolds, Cauchy-Riemann equations in symplectic cases. In these applications one of the key steps is to understand the compactness property of the corresponding moduli spaces of solutions. It has been well known from the important work of Sacks-Uhlenbeck [1] that the loss of compactness of a family of harmonic maps with uniformly bounded energy results from the fact the energy of these harmonic maps may concentrate at finitely many points on the manifold, at which bubblings occur. The same phenomenon occurs in gauge theory and Gromov's theory for symplectic manifolds.

Since M. Gromov introduced pseudo-holomorphic curves into the symplectic geometry in 1985 [2], the applications of pseudo-holomorphic curves to the symplectic

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geometry has become a main tool in the study of symplectic manifolds and achieved great success. Pseudo-holomorphic disks play a great role in the study of Lagrangian submanifolds in a symplectic manifold [3–6]. One key point in this application is the compactification of the space of pseudo-holomorphic curves. When the symplectic manifold, the chosen tamed and almost complex structure and the Lagrangian submanifold are smooth, the loss of compactness of a family of pseudo-holomorphic curves is well-known [5, 7–9], that is, this family may bubble off pseudo-holomorphic spheres and pseudo-holomorphic disks. In the case that the Lagrangian submanifold is not smooth, the corresponding result is not well established as indicated in [6]. In this paper we discuss the bubblings of a family of pseudo-holomorphic disks with uniformly bounded energy and with their boundaries in a totally-real subvariety L with only corner points as its singular points where L forms rational angles. In the smooth case, one needs the C^1-regularity both at interior points and at the boundary points to discuss the convergence in the C^1 topology. However in the case that the Lagrangian subvariety is not smooth, one should seek the solutions to the Cauchy-Riemann equation (suitably defined free boundary problem) in appropriate Sobolev spaces, so one faces the regularity problem, see [10] for the case that the subvariety is not smooth where the boundaries of pseudo-holomorphic disks lie on and one cannot expect the C^1-regularity at the singular boundary points. In this paper, we deform suitably the domain (in our case it is the unit disk D^2 in the plane) to resolve this problem.

2. Bubblings at the Corners

Let (M, ω) be a smooth symplectic manifold of dimension $2n$ with a smooth symplectic structure ω on M, and let $L_1, \cdots, L_N$ be smooth Lagrangian submanifolds of M. Let $J(\cdot)$ be a smooth almost complex structure on M, which is compatible with ω in the sense that $\omega(\cdot, J\cdot)$ defines a Riemann metric on M.

Fukuya and Oh [6] defined the following moduli space of pseudo-holomorphic disks in the case $M = T^*P$ for some smooth manifold P, ω is the standard symplectic structure on the cotangent bundle and L_i is the graph of the differential of some smooth function on P. Let p_i, $i = 1, \cdots, N$ be a fixed family of the transversal intersection points of L_i and L_{i+1} respectively. Here we assume $L_{N+1} = L_1$.

Fukuya and Oh [6] considered the following moduli space

$$\mathcal{M}_{J,\widetilde{L},\widetilde{p}} = \{u : D^2 \to M : u \text{ is } J-\text{holomorphic};\ u(z_i) = p_i,$$