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EXISTENCE OF TIME PERIODIC SOLUTIONS TO BOUNDARY VALUE PROBLEM OF ONE-DIMENSIONAL SEMILINEAR VISCOELASTIC DYNAMIC EQUATION WITH MEMORY*

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Dedicated to Professor Gu Chaohao on the occasion of his 70th birthday and his 50th year of educational work

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Abstract In this paper, we prove the existence of time-periodic solutions to the boundary value problem of semilinear one-dimensional dynamical equation for viscoelastic materials.

Key Words Nonlinear viscoelasticity; integrodifferential equation; periodic solution.

Classification 45K05, 73K15, 35B10.

1. Introduction

This paper concerns the study of the existence of time periodic classical solutions to boundary value problem for one dimensional viscoelastic equation

$$u_{tt}(x,t) = \frac{\partial}{\partial x}\sigma + f(x,t), \quad 0 < x < 1, \quad t \in \mathbf{R} \tag{1.1}$$

$$u(0,t) = u(1,t) = 0 \tag{1.2}$$

where the constitutive relation is given by

$$\sigma(t) = p(u_x(x,t)) - \int_0^\infty a(\tau) q(u_x(x,t-\tau))d\tau \tag{1.3}$$

and $p(\xi)$ is a linear function

$$p(\xi) = c^2\xi \tag{1.4}$$

In the past twenty years, there were a lot of works on the initial-boundary value problems of Equation (1.1), (1.3) (See, for instance, [1] and [2] for the existence of global

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classical solutions for small data; [3] for the existence of global weak solutions for large data). Nevertheless, there were only a few works dealing with periodic solutions to the boundary problem (1.1)–(1.3).

In the special case $p(\xi) \equiv q(\xi)$ and

$$p'(\xi) - \hat{a}(0)q'(\xi) > 0 \tag{1.5}$$

where $\hat{a}(0) = \int_0^\infty a(t)dt$, Freireisl [4] proved the existence of periodic weak solutions to the problem (1.1)–(1.3). Condition (1.5) implies that the material is viscoelastic solid. In linear viscoelasticity, we showed that for viscoelastic solid, the problem (1.1)–(1.3) has a T-periodic solution for any T-periodic function $f(x,t)$; and for a viscoelastic liquid like material, the problem admits a T-periodic solution for a T-periodic function $f(x,t)$ if and only if

$$\int_0^T f(x,t)dt = 0 \tag{1.6}$$

(See [5]).

In the present paper, we discuss the semilinear case where the material is viscoelastic solid, i.e. $p(\xi)$ has the form (1.4) and

$$c^2 - \hat{a}(0)q'(\xi) > 0 \tag{1.7}$$

In [6], Hrusa showed the global existence of classical solution to the initial-boundary value problem with large data and gave an estimation of the solution to the problem with kernel $a(t) = \exp(-\lambda t)$. Under some assumptions, we prove the existence of T-periodic classical solution to the problem (1.1)–(1.4) with a more general kernel $a(t)$.

2. Main Results

Throughout this paper, we assume that q and the kernel a satisfy the following hypotheses:

(H$_1$) $q \in C^1(\mathbf{R})$
with

$$q(0) = 0 \tag{2.1}$$

and there exist positive constants λ and μ such that

$$\mu \le \hat{a}(0)q'(\xi) \le c^2 - \lambda \tag{2.2}$$

(H$_2$) $a \in L^1(0,\infty) \cap C^\infty([0,\infty))$, $a(t) \not\equiv 0$ and there exists a positive constant δ_0 such that $b(t) = e^{\delta_0 t}a(t)$ is completely monotone (See, for example, [7]), that is

$$(-1)^j b^{(j)}(t) \ge 0, \quad j = 1, 2, \cdots$$