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REMARKS ON LOCAL REGULARITY FOR TWO SPACE DIMENSIONAL WAVE MAPS

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Dedicated to Professor Gu Chaohao on the occasion of his 70th birthday and his 50th year of educational work

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Abstract In this paper, we continue to study the equation

$$\square\phi^I + f^I(\phi, \partial\phi) = 0$$

where $\square = -\partial_t^2 + \Delta$ denotes the standard D'Alembertian in R^{2+1} and the nonlinear terms f have the form

$$f^I = \sum_{JK} \Gamma^I_{JK}(\phi) Q_0(\phi^J, \phi^K)$$

with

$$Q_0(\phi, \varphi) = -\partial_t\phi\partial_t\varphi + \sum_{i=1}^{2} \partial_i\phi\partial_i\varphi$$

and $\Gamma^I_{JK}(\phi)$ being C^∞ function of ϕ. In Y. Zhou [1], we showed that the initial value problem

$$\phi(0, x) = \phi_0(x), \quad \partial_t\phi(0, x) = \phi_1(x)$$

is locally well posed for

$$\phi_0 \in H^{s+1}, \quad \phi_1 \in H^s$$

with $s = \dfrac{1}{8}$. Here, we shall further prove that the initial value problem is locally well posed for any $s > 0$.

Key Words Wave equation; local well-posedness.
Classification 35L.

1. Introduction

In this paper, we continue to study the equation

$$\square\phi^I + f^I(\phi, \partial\phi) = 0 \tag{1.1}$$

where $\Box = -\partial_t^2 + \Delta$ denotes the standard D'Alembertian in R^{2+1} and the nonlinear terms f have the form

$$f^I = \sum_{JK} \Gamma^I_{JK}(\phi) Q_0(\phi^J, \phi^K) \tag{1.2}$$

with

$$Q_0(\phi, \varphi) = -\partial_t\phi\partial_t\varphi + \sum_{i=1}^{2} \partial_i\phi\partial_i\varphi \tag{1.3}$$

and $\Gamma^I_{JK}(\phi)$ being C^∞ function of ϕ. We call it the equations of wave maps type.

We are interested in the problem of minimal regularity of initial conditions for which the initial value problem

$$\phi(0, x) = \phi_0(x), \quad \partial_t\phi(0, x) = \phi_1(x) \tag{1.4}$$

is locally well posed. In Y. Zhou [1], we showed that the problem is locally well posed for

$$\phi_0 \in H^{s+1}, \quad \phi_1 \in H^s \tag{1.5}$$

with $s = \frac{1}{8}$. Here, we shall improve it to allow $s > 0$.

Theorem 1.1 *The initial value problem* (1.4) *for the equation* (1.1) *is locally well posed for* $\phi_0 \in H^{s+1}$ *and* $\phi_1 \in H^s$ *for any* $s > 0$.

In Section 2, we will state and prove a more precise version of Theorem 1.1.

2. Proof of Theorem 1.1

We begin with introducing a space-time norm similar to that in our previous paper [2]. We rewrite (1.1) as a first order system by letting

$$\phi_\pm = (\partial_t \mp \sqrt{-1}|D_x|)\phi \tag{2.1}$$

where

$$|D_x| = \sqrt{-\Delta} \tag{2.2}$$

then

$$(\partial_t \pm \sqrt{-1}|D_x|)\phi_\pm = f \tag{2.3}$$

Introduce the Fourier integral operators $F_\pm$ by

$$F_\pm\phi(t, x) = (2\pi)^{-2} \int e^{\sqrt{-1}(x\cdot\xi \pm t|\xi|)} \hat{\phi}(t, \xi) d\xi \tag{2.4}$$

Here and hereafter, $\hat{\phi}$ denotes the space Fourier transform of ϕ, then it follows from (2.3) that

$$\partial_t F_\pm \phi_\pm = F_\pm f \tag{2.5}$$