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CONVERGENCE OF ITERATIVE DIFFERENCE SCHEMES FOR TWO AND THREE DIMENSIONAL NONLINEAR PARABOLIC SYSTEMS*

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Abstract In this paper, we study the general difference schemes of the boundary value problem for the nonlinear parabolic systems with two and three space dimensions. To solve the nonlinear difference schemes, we construct an iterative sequence from the solutions of the linearized difference schemes. We shall prove the convergence of the difference solutions for the iterative difference schemes to the solution of the original boundary value problem of the nonlinear parabolic systems.

Key Words Difference scheme; two and three dimensional nonlinear parabolic systems; iteration.

Classification 65N10.

1. Introduction

1. In this paper, we are going to study the convergence of the iterative difference schemes for the boundary value problem of the nonlinear parabolic systems with two and three space dimensions.

In the case of two space dimensions, the following problem is considered

$$u_t = A(x,y,t,u,u_x,u_y)(u_{xx}+u_{yy}) + f(x,y,t,u,u_x,u_y) \tag{1}$$
$$(0 < x < l_1, 0 < y < l_2, 0 < t \le T)$$

$$\begin{aligned} u(0,y,t) &= \psi_1(y,t), \quad u(l_1,y,t) = \bar{\psi}_1(y,t) \\ u(x,0,t) &= \psi_2(x,t), \quad u(x,l_2,t) = \bar{\psi}_2(x,t) \end{aligned} \tag{2}$$

$$u(x,y,0) = \varphi(x,y) \tag{3}$$

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where $u(x,y,t) = (u_1(x,y,t), u_2(x,y,t), \cdots, u_m(x,y,t))$ is an m-dimensional vector unknown function ($m \geq 1$), $A(x,y,t,u,p_1,p_2)$ is a given $m \times m$ matrix function, and $f(x,y,t,u,p_1,p_2)$ is an m-dimensional vector function and $u_x = \frac{\partial u}{\partial x}$, $u_y = \frac{\partial u}{\partial y}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$, $u_{yy} = \frac{\partial^2 u}{\partial y^2}$ and $u_t = \frac{\partial u}{\partial t}$ are the corresponding m-dimensional vector derivatives of the m-dimensional unknown vector function $u(x,y,t)$, and $Q_T = \{0 \leq x \leq l_1, 0 \leq y \leq l_2, 0 \leq t \leq T\}$, $\psi_1(y,t), \bar{\psi}_1(y,t), \psi_2(x,y), \bar{\psi}_2(x,t)$ and $\varphi(x,y)$ are the given m-dimensional vector functions of variables $x \in [0,l_1]$, $y \in [0,l_2]$ and $t \in [0,T]$ respectively, with $l_1, l_2 > 0$ and $T > 0$.

2. Suppose that the following conditions for the problem (2) and (3) of the nonlinear parabolic system (1) are satisfied.

(I_2) The boundary value problem (2) and (3) for the nonlinear parabolic system (1) with two space variables has a unique smooth m-dimensional vector solution $u(x,y,t) \in C^{(4,2)}(Q_T)$.

(II_2) For the $m \times m$ coefficient matrix $A(x,y,t,u,p_1,p_2)$ there is a positive constant $\sigma_0 > 0$, such that for any $\xi \in \mathbf{R}^m$ and $(x,y,t) \in Q_T \equiv \{0 \leq x \leq l_1, 0 \leq y \leq l_2, 0 \leq t \leq T\}$ and $u, p_1, p_2 \in \mathbf{R}^m$,

$$(\xi, A(x,y,t,u,p_1,p_2)\xi) \geq \sigma_0 |\xi|^2 \tag{4}$$

is valid.

(III_2) The $m \times m$ matrix function $A(x,y,t,u,p_1,p_2)$ and the m-dimensional vector function $f(x,y,t,u,p_1,p_2)$ are continuous with respect to the variables $(x,y,t) \in Q_T$ and continuously differentiable with respect to the vector variables $u, p_1, p_2 \in \mathbf{R}^m$.

(IV_2) The boundary vector functions $\psi_1(y,t), \bar{\psi}_1(y,t), \psi_2(x,t), \bar{\psi}_2(x,t)$ have the continuous derivatives of second order with respect to space variables $(x,y) \in \Omega = \{0 \leq x \leq l_1, 0 \leq y \leq l_2\}$ and the continuous derivative of first order with respect to the time variable $t \in [0,T]$. And they satisfy the following conditions

$$\psi_1(0,t) = \psi_2(0,t), \quad \psi_2(l_1,t) = \bar{\psi}_1(0,t)$$
$$\bar{\psi}_1(l_2,t) = \bar{\psi}_2(l_1,t), \quad \bar{\psi}_2(0,t) = \psi_1(l_2,t)$$

The initial vector function $\varphi(x,y)$ is continuously differentiable with respect to the variables (x,y). Furthermore the equations

$$\psi_1(y,0) = \varphi(0,y), \quad \bar{\psi}_1(y,0) = \varphi(l_1,y)$$
$$\psi_2(x,0) = \varphi(x,0), \quad \bar{\psi}_2(x,0) = \varphi(x,l_2)$$

hold on the lateral sides of the rectangular bottom $\Omega = \{0 \leq x \leq l_1, 0 \leq y \leq l_2\}$ of the 3-dimensional domain Q_T.