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THE LIMIT OF THE STEFAN PROBLEM WITH SURFACE TENSION AND KINETIC UNDERCOOLING ON THE FREE BOUNDARY

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Abstract In this paper we consider the Stefan problem with surface tension and kinetic undercooling effects, that is with the temperature u satisfying the condition

$$u = -\sigma K - \varepsilon V_n \quad \text{on the interface } \Gamma_t,\ \sigma, \varepsilon = \text{const.} \geq 0$$

where K and V_n are the mean curvature and the normal velocity of Γ_t respectively. In any of the following situations: (1) $\sigma > 0$ fixed, $\varepsilon > 0$; (2) $\sigma = \varepsilon \to 0$; (3) $\sigma \to 0$, $\varepsilon = 0$, we shall prove the convergence of the corresponding local (in time) classical solution of the Stefan problem.

Key Words Limit; Stefan problem; lower order terms; model problem; Fréchet derivative.
Classification 35K65, 35R35.

1. Introduction

Let $\Omega \subseteq \mathbb{R}^3$ be a given bounded annular domain with a boundary consisting of disjoint components Γ_1 and Γ_2, where Γ_1 is the outside boundary. Let surface $\Gamma_t \subset \Omega$ divide Ω into two connected subdomains $\Omega_j(t)$ $(j = 1, 2)$ so that $\partial\Omega_j = \Gamma_t \cup \Gamma_j$. The modified Stefan problem (see [1]) consists of a pair (u^i, Γ)-finding which satisfies:

$$u_t^i - k^i \Delta u^i = 0 \quad \text{in } Q^i \equiv \bigcup_{t>0} (\Omega_i(t) \times \{t\}), \quad i = 1, 2 \tag{1.1}$$

$$k^1 \partial_n u^1 - k^2 \partial_n u^2 = l V_n \quad \text{on } \Gamma \bigcup_{t>0} (\Gamma_t \times \{t\}) \tag{1.2}$$

$$u^1 = u^2 = -\sigma K - \varepsilon V_n \quad \text{on } \Gamma \tag{1.3}$$

$$u^i = g^i(x, t) \quad \text{on } \Gamma_i \times (0, T), \quad i = 1, 2 \tag{1.4}$$

$$u^i \big|_{t=0} = \phi^i(x) \quad \text{on } \Omega_i(0), \quad i = 1, 2 \tag{1.5}$$

where u^i $(i = 1, 2)$ are temperatures, k^1, k^2 are the thermal diffusivity coefficients, l is the latent heat, n is the vector normal to Γ_t (pointing into $\Omega_2(t)$), V_n is the normal velocity of the hypersurface Γ_t, K is the mean curvature of Γ_t, $\sigma > 0$ is the surface tension coefficient, and ε is a nonnegative constant (generally regarded as an adjustable parameter).

Chen & Reitich [1] proved the existence and uniqueness of local (in time) classical solution for the problem (1.1)–(1.5) ($\sigma > 0$, $\varepsilon > 0$) by the Contraction Mapping Theorem. Radkevich [2] proved the existence of local classical solution for the problem (1.1)–(1.5) in both $\sigma > 0$, $\varepsilon > 0$ and $\sigma > 0$, $\varepsilon = 0$ situations by the investigation of a model parabolic initial value problem in a half space with boundary conditions containing a derivative with respect to time. The model problem is obtained by "freezing" the coefficients of the Fréchet derivative of the nonlinear operator determined by the Stefan condition and neglecting the "lower order terms". Bazaliy & Degtyarev [3] proved the convergence of the local classical solution of the problem (1.1)–(1.5) in the situation (2) by the Contraction Mapping Theorem.

This paper is devoted to the study of the convergence of the local classical solution of the problem (1.1)–(1.5) in the situations (1)–(3). For convenience of statement, we only discuss the situation (1) in detial, which can be regarded as a representative. For the situations (2) and (3), we only give the conclusions and the "model problems". In the forthcoming paper [4], we shall discuss the convergence of Verigin problem with surface tension at the free boundary.

This paper is organized as follows: In Section 2, we shall parameterize the free boundary by the distance function ρ from the initial position of the free boundary. In Section 3, we construct an initial approximation and introduce the two key Lemmas 3.1 and 3.2, on which the main result is based. In Sections 4 and 5, we prove Lemma 3.2. In Section 4, by localization, Hanzawa changes of function (see [5]), and "freezing" the coefficients and neglecting the "lower order terms", we derive the model problem. In Section 5, we study the model problem by establishing some estimates of its kernel.

2. Formulation of the Problem

Let $\Gamma_j \in C^{6+\alpha}$ ($j = 0, 1, 2$, $0 < \alpha < 1$) be given, here $\Gamma_0 = \Gamma_t\,|_{t=0}$. Given $0 < T < +\infty$, we set $Q_{jT} = \Omega_j(0) \times (0, T)$, $j = 1, 2$; $\Gamma_{jT} = \Gamma_j \times [0, T]$, $j = 0, 1, 2$. As the time changes, the interface Γ_0 varies and forms a free boundary which will be diffeomorphic to Γ_0 as long as T is small enough. We shall parameterize the free boundary by the distance funtion ρ from Γ_0 (in $\mathbb{R}^3$) and denote in by $\Gamma_{\rho T}$. The corresponding space-time domain in $\mathbb{R}^3 \times \mathbb{R}$, which will be diffeomorphic to Q_{jT}, is denoted by $Q_{j\rho T}$. The problem (1.1)–(1.5) is to determine the free boundary $\Gamma_{\rho T}$ and function $u_\rho^j(x, t)$ in the regions $Q_{j\rho T}$ ($j = 1, 2$):

$$\partial_t u_\rho^j - k^j \Delta u_\rho^j(x, t) = 0 \quad \text{in } Q_{j\rho T} \tag{2.1}$$

$$k^1 \partial_n u_\rho^1 - k^2 \partial_n u_\rho^2 = l V_n \quad \text{on } \Gamma_{\rho T} \tag{2.2}$$

$$u_\rho^1 = u_\rho^2 = -\sigma K - \varepsilon V_n \quad \text{on } \Gamma_{\rho T} \tag{2.3}$$