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ATTRACTORS AND DIMENSIONS FOR DISCRETIZATIONS OF A GENERALIZED GINZBURG-LANDAU EQUATION

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Dedicated to Professor Gu Chaohao on the occasion of his 70th birthday and his 50th year of educational work

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Abstract In this paper, we discretize the generalized Ginzburg-Landau equations with the periodic boundary condition by the finite difference method in spatial direction. It is proved that for each mesh size, there exist attractors for the discretized systems. The bounds for the Hausdorff dimensions of the discrete attractors are obtained, and the various bounds are independent of the mesh sizes.

Key Words Discretizations; generalized Ginzburg-Landau equation; attractors.
Classification 35K57, 35B40, 35L70.

1. Introduction

In [1] Brand and Dissler proposed the following generalized Ginzburg-Landau equation

$$\begin{aligned}\partial_t u + \nu u_x =& \chi u + (\gamma_r + i\gamma_i)u_{xx} - (\beta_r + i\beta_i)|u|^2 u - (\delta_r + i\delta_i)|u|^4 u \\ &- (\lambda_r + i\lambda_i)|u|^2 u_x - (\mu_r + i\mu_i)u^2\overline{u}_x\end{aligned} \tag{1.1}$$

where γ_r, δ_r, and χ are positive constants, $i = \sqrt{-1}$, $\nu, \gamma_i, \beta_r, \beta_i, \delta_r, \delta_i, \lambda_r, \lambda_i, \mu_r$ and μ_i are real constants. In [2] the periodic initial value problem for Equation (1.1) with the periodic boundary condition

$$u(x+1,t) = u(x,t), \quad x \in \mathbf{R}^1, \ t \geq 0 \tag{1.2}$$

and the initial condition

$$u(x,0) = u_0(x), \quad x \in \mathbf{R}^1 \tag{1.3}$$

is considered. Many people studied this problem for $\nu = \delta_r = \delta_i = \lambda_r = \lambda_i = \mu_r = \mu_i = 0$. When $\beta_r < 0$, $\gamma_r = 0$, the solution for Equation (1.1) may be "blowing up", that was pointed in [3]. In [2] we prove that the existence of global smooth solution and the attractor for the periodic initial value problem (1.1) (1.2) (1.3), and give the upper estimate for Hausdorff and fractal dimensions of the attractor.

In this paper, we discretize the generalized Ginzburg-Landau equations with the periodic boundary condition by the finite difference method in spatial direction. It is proved that for each mesh size, there exist attractors for the discretized systems. The bounds for the Hausdorff dimensions of the discrete attractors are obtained, and the various bounds are independent of the mesh sizes.

In Section 2, the discretization of the generalized Ginzburg-Landau equation and a uniformly priori estimate are studied. In Section 3, the existence for the attractors for the discretized systems is obtained. In Section 4, the uniform upper bounds for the Hausdorff dimensions of attractors are therefore proved.

2. Discretization of Generalized GL Equations and *a Priori* Estimate

To study the spatially discretized generalized GL equation (1.1) with the initial periodic boundary condition (1.2) (1.3), we use the finite difference method.

Let $J \in N$, $h = \dfrac{1}{J}$. We approximate a function $u(x) \in L^2(0,1)$, real valued or complex valued, is set to be

$$u = (u_1, u_2, \cdots, u_J)^{tr} = \left(u\left(\frac{1}{J}\right), u\left(\frac{2}{J}\right), \cdots, u(1)\right)^{tr} \tag{2.1}$$

As usual, the discretized negative Laplacian operator $-\Delta$ with periodic boundary condition by using the finite difference scheme is set to be

$$A = J^2 A_1 = J^2 \begin{pmatrix} 2 & -1 & 0 & \cdots & \cdots & \cdots & -1 \\ -1 & 2 & -1 & \cdots & \cdots & \cdots & 0 \\ 0 & -1 & 2 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 2 & -1 & 0 \\ \cdots & \cdots & \cdots & \cdots & -2 & 2 & -1 \\ -1 & \cdots & \cdots & \cdots & 0 & -1 & 2 \end{pmatrix}_{J\times J} \tag{2.2}$$

the following notations are used

$$u_{jx} = \frac{1}{h}(u_{j+1} - u_j) = \frac{1}{h}\Delta_+ u_j, \quad u_{j\bar{x}} = \frac{1}{h}(u_j - u_{j-1}) = \frac{1}{h}\Delta_- u_j$$