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# A NOTE ON THREE CLASSES OF NONLINEAR PARABOLIC EQUATIONS WITH MEASURABLE COEFFICIENTS*

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Dedicated to the 70th birthday of Prof. Wang Rouhuai

**Abstract** In this note a noval and simple technique is presented to replace the complicated one in [1] to obtain Hölder continuity of the weak solutions for a class of nonlinear parabolic equations with measurable coefficients, whose prototype is the singular $p$-Laplacian. This new approach is also applied to two other classes of nonlinear parabolic equations with measurable coefficients, whose weak solutions exhibit the similar property to those of equations mentioned above.

**Key Words** Hölder continuity; nonlinear parabolic equations measurable coefficients.

**Classification** 35D10, 35K55.

## 1. Introduction

Let $\Omega$ denote a domain in $R^N$, and for $0 < T < \infty$ set $\Omega_T = \Omega \times (0, T)$. Functions $a^{ij}(x,t)\ (i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, N)$ are only assumed measurable and satisfy the ellipticity condition

$$\lambda_1|\xi|^2 \le a^{ij}\xi^i\xi^j \le \lambda_2|\xi|^2, \quad \forall \xi \in R^N, \text{ a.e. } (x,t) \in \Omega_T \tag{1.1}$$

for some $\lambda_1, \lambda_2 > 0$. We consider the following three classes of equations

(A) $$u_t - (a^{ij}(x,t)|Du|^{p-2}u_{x_i})_{x_j} = 0, \quad 1 < p \le 2$$

whose simpler form is the singular $p$-Laplacian equation

$$u_t - \operatorname{div}(|Du|^{p-2}Du) = 0, \quad 1 < p < 2 \tag{1.2}$$

(B) $$(\Phi(u))_t - (a^{ij}(x,t)|Du|^{p-2}u_{x_i})_{x_j} = 0, \quad 1 < p \le 1 + \frac{1}{m}$$

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where the function $\Phi : R^+ \to R^+$ satisfies, for $0 < s_1 \le s_2$,

$$\frac{1}{\Lambda} \le \frac{\Phi'(s_1)}{\Phi'(s_2)} \le \Lambda\Big(\frac{s_1}{s_2}\Big)^{\frac{1}{m}-1}, \quad m \ge 1, \Lambda \ge 1 \tag{1.3}$$

and $\Phi'(s) > 0, \forall s > 0$.

As a matter of fact, the equation of non-Newtonian polytropic filtration

$$u_t - \operatorname{div}(|Du^m|^{p-2} Du^m) = 0, \quad 1 < p < 1 + \frac{1}{m}, m > 1 \tag{1.4}$$

is its particular form.

(C) $$(\Phi(u))_t - (a^{ij}(x,t)u_{x_i})_{x_j} = 0$$

where function $\Phi : R^+ \to R^+$ satisfies, for $0 < s_1 \le s_2$,

$$\frac{1}{\Lambda}\Big(\frac{s_1}{s_2}\Big)^{\frac{1}{m}-1} \le \frac{\Phi'(s_1)}{\Phi'(s_2)} \le \Lambda, \quad 0 < m < 1, \Lambda \ge 1 \tag{1.5}$$

and $\Phi'(s) > 0, \forall s > 0$.

In fact, the equation of fast diffusion

$$u_t - \Delta u^m = 0, \quad 0 < m < 1 \tag{1.6}$$

is its prototype.

The weak solutions of Equations (A), (B) and (C) are included correspondingly in the following spaces

(A′) $$u \in C_{\text{loc}}(0,T; L^2_{\text{loc}}(\Omega)) \cap L^p_{\text{loc}}(0,T; W^{1,p}_{\text{loc}}(\Omega)) \cap L^\infty_{\text{loc}}(\Omega_T)$$

(B′) $$\Phi'(u) \in C_{\text{loc}}(0,T, L^1_{\text{loc}}(\Omega)), \quad u \in L^p_{\text{loc}}(0,T, W^{1,p}_{\text{loc}}(\Omega)) \cap L^\infty_{\text{loc}}(\Omega_T)$$

(C′) $$\Phi'(u) \in C_{\text{loc}}(0,T; L^1_{\text{loc}}(\Omega)), \quad u \in L^2_{\text{loc}}(0,T; W^{1,2}_{\text{loc}}(\Omega)) \cap L^\infty_{\text{loc}}(\Omega_T)$$

Virtually the local boundedness of the weak solutions may easily be proved under certain restrictions ([2, 3]). We notice that for Equation (1.4), the necessary and sufficient criterion of existence of FRPP (cf.[4]) (finite rate of propagation of perturbations) is

$$(p-1)m > 1, \quad \text{i.e.,} \quad p > 1 + \frac{1}{m} \tag{1.7}$$

In the meanwhile the same conclusion remains true for Equation (1.2) with $m = 1$ and Equation (1.6) with $p = 2$. When (1.7) is violated, due to the regularizing effect the non-negative solutions for Equations (A), (B) and (C) behave quite similar to the solutions of $u_t - \Delta u = 0$. The following theorem exhibits such behavior to a great extent.