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MIXED INITIAL BOUNDARY-VALUE PROBLEM FOR SOME MULTIDIMENSIONAL NONLINEAR SCHRÖDINGER EQUATIONS INCLUDING DAMPING *

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Abstract The motivation of this paper is the study of the unique existence of weak and smooth solutions for the mixed initial boundary-value problem of some multidimensional nonlinear Schrödinger equations including damping.
Key Words Global solutions; nonlinear Schrödinger equations; Galerkin method
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1. Introduction

The problem of nonlinear Schrödinger equations including growth damping have been studied in the paper [1–3], and the Chaos phenomenon has been found in these problems [4, 5].

The purpose of this article is to investigate the existence and uniqueness of solutions for the following mixed initial boundary-value problem

$$\frac{\partial \vec{u}}{\partial t} - \gamma \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\Big(a_{ij}(x)\frac{\partial \vec{u}}{\partial x_j}\Big) + b(x)q(|\vec{u}|^2)\vec{u} + C(x,t)\vec{u}$$

$$= \vec{f}(x,t), \quad x \in \Omega, t > 0 \tag{1.1}$$

$$\vec{u}(x,0) = \vec{u}_0(x), \quad x \in \Omega \tag{1.2}$$

$$\Big(\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial \vec{u}}{\partial x_i}\cos(\vec{n}, x_j) + h(x)\vec{u}\Big)\Big|_{\partial\Omega} = 0 \tag{1.3}$$

where $\vec{u} = (u_1(x,t), u_2(x,t), \cdots, u_N(x,t))$ is an unknown complex vector-value function, Ω is a bounded domain with boundary $\partial\Omega \in C^2$, $\vec{n}$ denotes the outward unit normal of $\partial\Omega$. On the complex functions $C(x,t) = (C_{ij}(x,t))_{N\times N}$, $\vec{f}(x,t) = (f_1(x,t), \cdots, f_N(x,t))$,

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and the real functions $a_{ij}(x)(i,j=1,\cdots,n), h(x), b(x), q(s)$, we make the following assumptions

$$\text{I)}\quad \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq a_0|\xi|^2,\ \forall x\in\Omega,\ \xi=(\xi_1,\cdots,\xi_n)\in R^n,\ a_0>0,$$

$$a_{ij}=a_{ji},\ a_{ij}(x)\in C^1(\bar\Omega).$$

$$\text{II)}\quad C_{ij}(x,t), \frac{\partial C_{ij}(x,t)}{\partial t}\in L^\infty(Q_T), (i,j=1,\cdots,N), Q_T=(0,T)\times\Omega.$$

$$\text{III)}\quad b(x)\geq 0,\ h(x)\geq 0,\ q(s)\geq 0,\ h(x)\in C^0(\bar\Omega),\ q(s)\in C^1(R^+),$$

$$b(x)\in C^0(\bar\Omega).$$

$$\text{IV)}\quad \vec f(x,t),\ \vec f_t(x,t)\in L^\infty(0,T;L^2(\Omega)),\ \vec u_0(x)\in H^2(\Omega),\ \gamma=\gamma_0+i\gamma_1,$$

$$\gamma_0\geq 0,\ |\gamma|>0.$$

We shall employ the Galerkin method to prove the existence of weak and smooth solutions of the problem (1.1)–(1.3).

Let $\{w_i(x)\}_{i=1,2,\dots}$ be the solutions of the eigenvalue problem

$$\begin{cases} -\sum\limits_{i,j=1}^{n}\dfrac{\partial}{\partial x_i}\Big(a_{ij}(x)\dfrac{\partial w_k}{\partial x_j}\Big)=\lambda_k w_k \\ \Big(\sum\limits_{i,j=1}^{n}a_{ij}(x)\dfrac{\partial w_k}{\partial x_i}\cos(\vec n,x_j)+hw_k\Big)\Big|_{\partial\Omega}=0 \end{cases}$$

Then one can see that $\{w_i(x)\}$ is a complete system of functions in $H^2(\Omega)$. We look for an approximate solutions $\vec u_m(x,t)$ in the form

$$\vec u_m(x,t)=\sum_{k=1}^{m}\vec\alpha_{km}(t)w_k(x) \tag{1.4}$$

where $\vec u_m=(u_{m_1}(x,t),\cdots,u_{mN}(x,t))$. The unknown complex functions $\vec\alpha_{km}(t)=(\alpha_{km1}(t),\cdots,\alpha_{kmN}(t))$ are determined by the following system of ordinary differential equations

$$\begin{aligned}(u_{mlt},w_s)+\gamma a(u_{ml},w_s)+(b(x)q(|\vec u_m|^2)u_{ml},w_s)\\ +\Big(\sum_{j=1}^{N}C_{lj}u_{mj},w_s\Big)=(f_l,w_s)-\gamma\int_{\partial\Omega}hu_{ml}w_s ds\end{aligned} \tag{1.5}$$