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REFLECTION OF SINGULARITIES AT BOUNDARY FOR PIECEWISE SMOOTH SOLUTIONS TO SEMILINEAR HYPERBOLIC SYSTEMS

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1. Introduction

During the last decade, much has been done in the study of singularity propagation for nonlinear partial differential equations, which has already been introduced in [1] and [6]. However, one has not seen much work on the reflection of singularities at boundary before. In 1979, M. Reed & J. Berning [8] proved that the singularities still propagate along the characteristic curves after reflection at the boundary for semilinear wave equations in one-dimensional case. For the multi-dimensional case, there have been a lot of work lately, such as that done by G. Métivier [6], M. Beals & G. Métivier [2] and M. Sablé-Tougeron [9], who use the tools of pseudodifferential and paradifferential operators in conormal distributions. Nevertheless, one has not seen any work done by classical methods in piecewise smooth solutions up till now, while to solve this problem is of equal importance.

On the other hand, J. Rauch & M. Reed [7] proved the following result about the singularity propagation of the solution to the Cauchy problem for semilinear hyperbolic systems by classical methods:

Given a symmetric strictly hyperbolic system

$$\begin{aligned}\partial_t u(t, x) + \sum_{j=1}^{n} A_j(t, x)\,\partial_{x_j} u(t, x) + B(t, x)\,u(t, x) &= (Pu)(t, x) \\ &= f(u;\ t,\ x), \qquad x \in \Omega \qquad (1.1)\end{aligned}$$

where $x = (x_1, x_2, \dots, x_n) = (x_1, x')$; P is the partial differential operator representing the first line of the equations; $A_j's$ are 2×2 symmetric matrices, B is a 2×2 matrix, $u = (u_1, u_2)^T$; $A_j(t, x)'s$ and $B(t, x)$ are smooth enough, $f(u; t, x)$ is smooth enough with respect to its arguments; $A_j's$, B and f are all constants outside a compact set with respect to (t, x). If the initial data are piecewise smooth with a jump discontinuity across an $(n$

— 1) -dimensional hypersurface σ, then there is a unique local piecewise smooth solution to the Cauchy problem of (1. 1), whose singularities propagate along the two characteristic hypersurfaces issuing from σ. After that, Chen Shuxing proved the same result in the 3×3 case in [5].

In this paper, one has proved the following main theorem:

Theorem. *For (1. 1) and a boundary condition*

$$Mu|_{\partial\Omega}=0 \qquad (1.2)$$

where $M\in C(\partial\Omega:\ Hom\ R^2)$, the boundary $\partial\Omega$ of the region Ω is regular, (1. 2) is a stably admissible boundary condition (cf. [11], [12]), $[0, T]\times\partial\Omega$ is noncharacteristic for small T. If in $t\leq 0$, there is a piecewise smooth solution to (1. 1) and (1. 2) whose singularities propagate along a characteristic hypersurface $\sum_2$ where $\sum_2\cap\{t\geq 0\}$ intersects with the boundary $\partial\Omega\times[0, T]$ at an $(n-1)$ -dimensional hypersurface σ and is reflected into another characteristic hypersurface $\sum_1$; then in $t\geq 0$, there is still a local piecewise smooth solution whose singularities propagate along $\sum_1\cup\sum_2$. Furthermore, the order of the singularities is maintained after the reflection.

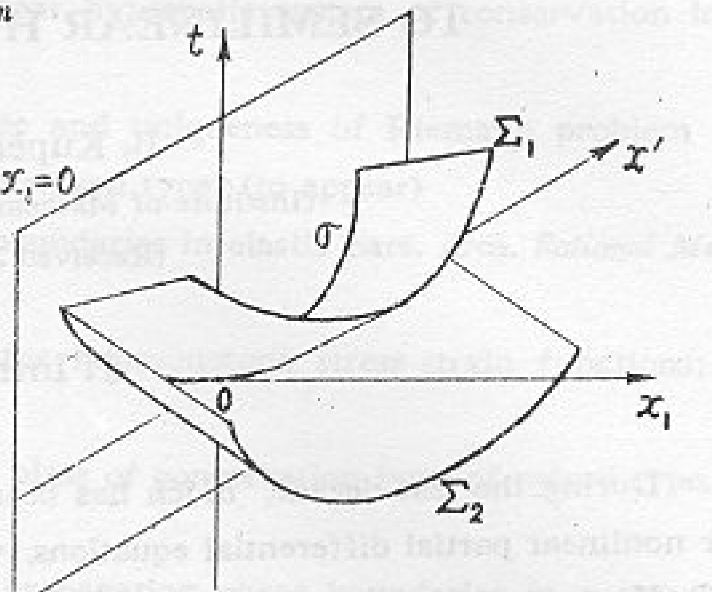

Fig. 1

In our paper, we will first simplify the problem in section 2, and then we will estimate the jumps across the characteristic hypersurface $\sum_1$. The proof of the main theorem will be given in section 4 and the order of the singularities will be discussed in section 5.

2. Simplification

Thanks to the regularity of the boundary, we can flatten it locally by a transformation of the independent variables. So we suppose the boundary to be $\{x_1=0\}$ and O to be the origin, as shown in Fig. 1. We suppose further that $\sum_1$ is above $\sum_2$, where $\sum_i (i=1, 2)$ is the characteristic hypersurfaces corresponding to the eigenvalues $\lambda_i (i=1, 2)$, that is to say, $\lambda_2<0<\lambda_1$. First, we must solve the problem in $I=\{(t, x)\,|\,t\geq 0,\ (t, x) \text{ is below } \sum_2\}$.

Denote σ_2 to be the $(n-1)$ -dimensional hypersurface at which $\sum_2$ intersects with the initial plane. σ_2 is tangent to the boundary at O. The solution is known in the region