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Adaptive Surface Reconstruction Based on Tensor Product Algebraic Splines

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Abstract. Surface reconstruction from unorganized data points is a challenging problem in Computer Aided Design and Geometric Modeling. In this paper, we extend the mathematical model proposed by Jüttler and Felis (Adv. Comput. Math., 17 (2002), pp. 135-152) based on tensor product algebraic spline surfaces from fixed meshes to adaptive meshes. We start with a tensor product algebraic B-spline surface defined on an initial mesh to fit the given data based on an optimization approach. By measuring the fitting errors over each cell of the mesh, we recursively insert new knots in cells over which the errors are larger than some given threshold, and construct a new algebraic spline surface to better fit the given data locally. The algorithm terminates when the error over each cell is less than the threshold. We provide some examples to demonstrate our algorithm and compare it with Jüttler's method. Examples suggest that our method is effective and is able to produce reconstruction surfaces of high quality.

AMS subject classifications: 65D17

Key words: Surface reconstruction, algebraic spline surface, adaptive knot insertion.

1. Introduction

With the development of modern industry, it is possible to capture extremely large unorganized data points from the surfaces of existing models and products. Meanwhile, reproduction of existing models and products with complex free-form surfaces plays a very important role in CAD/CAM, Computer Vision, Computer Graphics, etc. The significance of surface reconstruction from point clouds attracts many researchers to investigate efficient and robust algorithms to solve the problem.

Surface reconstruction has been widely studied since the 1980s. A class of approaches in parametric surface reconstruction are based on the active contour models which were first proposed in [12] to detect image contours. Pottmann et al. [15] applied the technique to surface approximation, and they proposed an active parametric B-spline model

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to fit smooth given curves and surfaces [14]. Wang et al. [21] extended their work to the case of unorganized point cloud data and improved the efficiency of Pottmann's method dramatically. Recently, an evolution-based least square fitting method was also put forward to fit point clouds [2] and curves [18]. However, parametric fitting methods are difficult to handle point clouds with complicated topology. And also, parametric surface reconstruction needs a process of parametrization which is a non-trivial problem.

To solve the difficulty of parametric curve/surface reconstruction, implicit representation has been introduced. Carr et al. [4] introduced polyharmonic radical basis functions and multi-pole methods to model large data sets by a single radical basis function. Zhao et al. [26,27] applied the level set method in surface reconstruction by solving a PDE equation numerically. Their approach becomes very expensive both in time and in usage of memory when high accuracy reconstruction is required. Alexa et al. [1] developed the projection-based approaches, which have the advantage that they are local and they directly yield a point on the surface. Their approach requires the solution of a non-linear moving least square problem in a projection set, which makes many geometrical operations expensive.

The signed distance function has been used to reconstruct an implicit surface on a rectangular grid with the signs to distinguish inside and outside of the surface [3, 7, 8]. Ohtake et al. [13] proposed a hierarchical approach for 3D scattered data interpolation with compactly supported basis functions. Although this approach can process very large set of points, the implicit function used does not have an explicit form which is critical to theoretical analysis, such as multi-resolution analysis, approximation error, etc. Jüttler and Felis [10] described a technique for fitting surfaces to scattered data by simultaneously approximating points and associated normal vectors which are estimated from the given data. This approach is quite efficient due to a simple representation using algebraic B-spline functions with fixed meshes and a linear optimization method. However, the method can not get satisfactory result when the data points contain rich local geometric details. Other work includes moving least squares [6], dynamic implicit surface reconstruction [22, 24, 25], etc.

In this paper, we extend the mathematical model proposed by Jüttler and Felis in [10] from fixed tensor-product meshes to adaptive tensor-product meshes. The basic idea is as follows. We start with an initial mesh, over which the model proposed by Jüttler is applied to obtain an initial fit to the given point cloud. Then we check the fitting errors over each cell, and insert new knots in cells over which the errors are larger than some given threshold, and reconstruct a new algebraic surface to locally fit the given data. This process is recursively applied until the errors over each cell of the mesh are less than the given threshold. Our approach can produce reconstruction surfaces with much higher quality and is much more efficient than Jüttler's method due to adaptive meshes generation.

The rest of the paper is organized as follows. In Section 2, the algebraic B-spline surfaces are briefly introduced. In Section 3, we review the optimization reconstruction model proposed by Jüttler and Felis in [10] and extend it to adaptive meshes in the next section. In Section 5, we demonstrate our new algorithm with some examples and compare it with the technique presented in [10]. Finally, we conclude the paper by proposing some problems for future research in Section 6.