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A Second-Order Method for the Electromagnetic Scattering from a Large Cavity

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Abstract. In this paper, we study the electromagnetic scattering from a two dimensional large rectangular open cavity embedded in an infinite ground plane, which is modelled by Helmholtz equations. By introducing nonlocal transparent boundary conditions, the problem in the open cavity is reduced to a bounded domain problem. A hypersingular integral operator and a weakly singular integral operator are involved in the TM and TE cases, respectively. A new second-order Toeplitz type approximation and a second-order finite difference scheme are proposed for approximating the hypersingular integral operator on the aperture and the Helmholtz in the cavity, respectively. The existence and uniqueness of the numerical solution in the TE case are established for arbitrary wavenumbers. A fast algorithm for the second-order approximation is proposed for solving the cavity model with layered media. Numerical results show the second-order accuracy and efficiency of the fast algorithm. More important is that the algorithm is easy to implement as a preconditioner for cavity models with more general media.

AMS subject classifications: 78M20, 65N22, 65N06

Key words: Electromagnetic scattering, Helmholtz equation, fast algorithm, Toeplitz matrix, second-order method.

1. Introduction

Electromagnetic scattering is one of the most competitive areas in both mathematical and engineering communities comprising of wide range of applications, such as radar, remote sensing, geoelectromagnetics, bioeletromagnetics, antennas, wireless communication, optics and high-frequency/high-speed circuits. In this paper, we are mainly concerned with the electromagnetic scattering from a two-dimensional large open cavity embedded in an infinite ground plane. The geometry of the cavity is shown in Fig. 1. We assume that the ground plane and the walls of the open cavity are perfect electric conductors (PEC), and the interior of the open cavity is filled with non-magnetic materials which may be inhomogeneous. The half-space above the ground plane is filled with a homogeneous, linear

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and isotropic medium. In this setting, the electromagnetic scattering by the cavity is governed by the Helmholtz equations along with Sommerfeld's radiation conditions imposed at infinity.

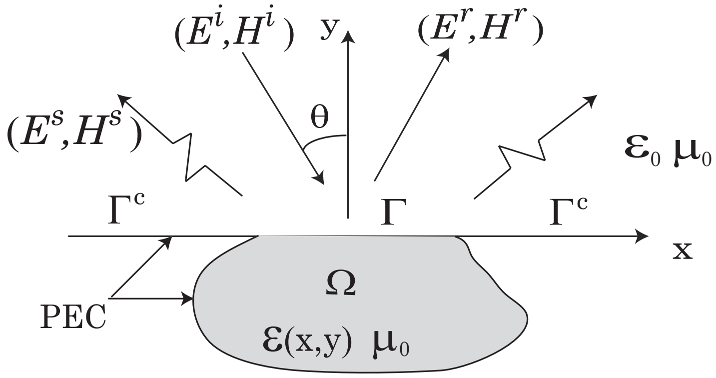

Figure 1: The geometry of the cavity.

Recently, the large cavity problem has attracted much attention because of its significant industrial and military applications. Examples of cavities include jet engine inlet ducts, exhaust nozzles and cavity-backed antennas. The Radar Cross Section (RCS) is an important physical parameter that characterizes scattering by a cavity. Therefore, accurate prediction of the RCS of the cavity is very necessary due to its dominance to the target's overall RCS. However, the accurate computation is especially difficult due to the highly oscillatory nature of the fields when the cavity is large compared to the wavelength of the fields. One often uses finer meshes or higher-order numerical approximations to achieve the better accuracy. In this paper, we intend to develop a second-order method for cavity models and corresponding fast algorithms when the medium inside the rectangular cavity is vertically layered.

In many practical applications, one is interested in the cavity problem with either a large wavenumber k or a large diameter a of the computational domain, which leads to the large "ka" numbers. A straight-forward change of coordinates yields the equivalence of large wavenumbers and large cavity problems. For convenience, without loss of generalities, we focus primarily on large wavenumber problems in our discussion. There are several difficulties for solving the problem with a large wavenumber. One lies in the fact that the solution for a large wavenumber is highly oscillatory. Also, it is well known that error estimates strongly depend upon the wavenumber. Babuska and Sauter [4] showed that for a related model problem, the ratio of the error of the Galerkin solution and the error of the best approximation tends to infinity as the wavenumber increases. Aziz et al. [3] pointed out that the condition "k^2h small" would be required to ensure that the error of the linear FEM solution has the same magnitude as the error of the best approximation, where h is the mesh size. Moreover, approximations of models with large wavenumbers always result in a large, sparse, symmetric, non-Hermitian, indefinite and ill-conditioned discrete system, for which direct methods are extremely expensive and classical iterative algorithms are slowly convergent or even fail to converge.