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Boundary Conditions for Limited Area Models Based on the Shallow Water Equations

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Abstract. A new set of boundary conditions has been derived by rigorous methods for the shallow water equations in a limited domain. The aim of this article is to present these boundary conditions and to report on numerical simulations which have been performed using these boundary conditions. The new boundary conditions which are mildly dissipative let the waves move freely inside and outside the domain. The problems considered include a one-dimensional shallow water system with two layers of fluids and a two-dimensional inviscid shallow water system in a rectangle.

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1 Introduction

The problem of boundary conditions in a limited domain is recognized as an important problem in geophysical fluid dynamics. In its primary form, the problem which was

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identified already by J. Von Neumann and J. Charney, is to choose the boundary conditions for a limited area model (LAM), when the boundary of the computational domain or part of it has no physical relevance and there are no physical laws prescribing a natural boundary condition. This old problem which persists in this form, reappears in other contemporary developments. For multi-level numerical methods, the issue of boundary conditions is present for each of the subdomains of the fine grids; see e.g. [24]. Also, in many attempts at modeling, this issue occurs as well, the usual remedy being to use the space periodic boundary conditions. However space periodicity can be physically unrealistic in particular because it generally implies conservation of energy; see e.g. [31] for the modeling of clouds where the lack of suitable boundary conditions prevents from studying the interactions of contiguous vertical columns.

Many authors have addressed the problem from different angles, but the approach is generally based on some form of physical intuition or on some modeling; see e.g. [5–7, 22, 24–26, 28, 32, 36, 37], the tutorial [38] and the references therein. In this article we would like to address this issue in the light of theoretical (mathematical) results connected to the study of the well-posedness of the initial and boundary value problems. The problems studied here are the two-layer shallow water equations, in space dimension one and the (one layer) shallow water equations in a rectangle in space dimension two; see [30] for the one-dimensional two-layer shallow water equations; see also [29] for the one-dimensional (one layer) shallow water equations; and see [8, 20, 21] for the general approach to initial and boundary value problems for hyperbolic equations based on the so-called Kreiss-Lopatinsky conditions; see also [23].

Beside presenting these boundary conditions, another aim of this article is to report on numerical tests performed using these conditions. One feature of these boundary conditions is that they are (mildly) dissipative, a property which is used of course in the theoretical study. In our numerical studies these boundary conditions appear to let the waves move freely inside or outside the domain. In the mathematical literature such boundary conditions are called "transparent" boundary conditions; see e.g. [11, 12, 14]. Note that our objective per se is not to derive transparent boundary conditions, but it happens that the boundary conditions coming from the existence and uniqueness theorem are transparent to a certain extent.

The first example on which we report in Section 2 is that of two layers of inviscid shallow water equations in space dimension one. Note that this case studied in [25] and from the mathematical perspective in [30] raises substantial new difficulties as compared to the case of one layer. Indeed this system may not be hyperbolic, see e.g. [3, 9, 25] and also [10,33,37] and as emphasized already in [27] the boundary conditions for multilayer shallow water equations can not be of local type, which means in the present case, that on each layer, the boundary conditions possibly involve the other layer(s). The boundary conditions that we discuss below are studied on the theoretical side in [30] and on the computational side they appear to be transparent to some extent.

The second case that we test and discuss in Section 3 is that of the inviscid shallow water equations in a rectangle in space dimension two. Note that here there is no sup-