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The Convex Relaxation Method on Deconvolution Model with Multiplicative Noise

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Abstract. In this paper, we consider variational approaches to handle the multiplicative noise removal and deblurring problem. Based on rather reasonable physical blurring-noisy assumptions, we derive a new variational model for this issue. After the study of the basic properties, we propose to approximate it by a convex relaxation model which is a balance between the previous non-convex model and a convex model. The relaxed model is solved by an alternating minimization approach. Numerical examples are presented to illustrate the effectiveness and efficiency of the proposed method.

AMS subject classifications: 52A40, 65K10, 65K15, 90C26

Key words: Alternating minimization, convergence, deblurring, multiplicative noise, non-convex model.

1 Introduction

The electromagnetic field is a physical field emitted by electrically charged objects and which impacts the properties of charged objects in the neighborhood of the field. Visible light is the electromagnetic field in certain range of frequencies. The electromagnetic field always bears important information of the charged objects generating it. This is the starting point of imaging formation. However, as the propagation of wave field usually suffers modest blurring, the loss of information seems to be unavoidable. Moreover, the observed field can be further affected by noise due to many known or unknown factors. In real applications, similar problems always exist. For instance, the photos of space targets produced by astronomical telescopes are often blurred by atmospheric

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turbulence. In these regards, image blur and noise are fundamental problems in the domain of image processing and they continue to attract the attention of researchers.

Evidently, the deblurring process under noise is not well-posed in the sense of Hadamard [13]. The observed blurred and noisy image only provides partial restrictions on the solution image. There exist various candidature images which can match the observed degraded image under the given blur operator. Hence, the greatest challenge in deblurring and denoising is to design methods for restoring solutions toward more reasonable results adaptive to certain prior information [19].

In order to reconstruct the image, we need a mathematical description to show how the noisy and blurry image is formed. If that is not available, there are many algorithms to estimate the blur though it is beyond the scope of the current papers. In literatures, one usually supposes that the degraded image $\boldsymbol{f}$ is formed as:

$$\boldsymbol{f}=\boldsymbol{H}\boldsymbol{w}+\boldsymbol{v}, \tag{1.1}$$

here $\boldsymbol{f}$, $\boldsymbol{w}$ and $\boldsymbol{v}$ are an mn-by-1 vector corresponding to an m-by-n image, and they are observed image, original image and additive noise respectively. And the matrix $\boldsymbol{H}\in\mathbb{R}^{mn\times mn}$ is the blur operator. Without loss of generality, here we assume that $\boldsymbol{H}\geq 0$ holds in component-wise meaning.

Under this additive noise scheme, various approaches have been studied to handle the inverse problem (1.1). Among them, regularization methods which propose to find a solution image satisfying the previous mentioned prior information seem to be rather appealing. Many regularization techniques are established by least-square methods. As the number of unknowns concerned might be huge, these approaches are commonly iterative in nature. In the past decades, the total variation (TV) regularization functional has become popular since it can efficiently reserve sharp edges of images, see [1, 3, 8, 22, 27, 28]. This type of algorithm minimizes a functional composing of two different terms. One term is restricted on the blur operator and the other term corresponds to the TV regularization for the solution image.

Note that there have been a great deal of deblurring approaches in literatures devoting to removing the additive noise. In recent years, in a different discipline, much attention also has been paid increasingly to many other kinds of random noises. Such noises include multiplicative noise, see for instance [2, 5, 14, 21]; impulse noise and Poisson noise etc., see for instance [4, 6, 7, 15, 26] and [9, 20]. In this paper, we focus on the deblurring issues under the multiplicative noise.

The image reconstruction problem under multiplicative noise is evidently challenging since this kind of noise contaminates images in a totally different way from additive noise. That is, the observed image $\boldsymbol{f}$ is supposed to be the multiplication of the blurred image $\boldsymbol{H}\boldsymbol{w}$ and the noise $\boldsymbol{v}$:

$$\boldsymbol{f}=\boldsymbol{H}\boldsymbol{w}\boldsymbol{v}, \tag{1.2}$$

where again, $\boldsymbol{H}$ is the blur operator and $\boldsymbol{w}$ is the clean image. Note that here the noise $\boldsymbol{v}$ could follow different statistics such as Gaussian, Gamma or other distributions. Without