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AUSM-Based High-Order Solution for Euler Equations

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Abstract. In this paper we demonstrate the accuracy and robustness of combining the advection upwind splitting method (AUSM), specifically AUSM$^+$-UP [9], with high-order upwind-biased interpolation procedures, the weighted essentially non-oscillatory (WENO-JS) scheme [8] and its variations [2,7], and the monotonicity preserving (MP) scheme [16], for solving the Euler equations. MP is found to be more effective than the three WENO variations studied. AUSM$^+$-UP is also shown to be free of the so-called "carbuncle" phenomenon with the high-order interpolation. The characteristic variables are preferred for interpolation after comparing the results using primitive and conservative variables, even though they require additional matrix-vector operations. Results using the Roe flux with an entropy fix and the Lax-Friedrichs approximate Riemann solvers are also included for comparison. In addition, four reflective boundary condition implementations are compared for their effects on residual convergence and solution accuracy. Finally, a measure for quantifying the efficiency of obtaining high order solutions is proposed; the measure reveals that a maximum return is reached after which no improvement in accuracy is possible for a given grid size.

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Key words: Shock capturing, advection upwind splitting, Euler equations, weighted essentially non-oscillatory, monotonicity preserving.

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1 Introduction

The complete procedure for solving time-dependent partial differential equations of conservations laws with upwind schemes in the finite-volume setting consists of three basic

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steps: reconstruction, flux evaluation, and time integration. The boundary condition is included as part of the flux evaluation before the solution in the whole domain is time-advanced. There have been enormous studies done for improving methodologies in each step. As each step provides the data needed for the next step, it is expected that different results should surface for different combinations of methods for each step. In recent years high order interpolations have received increased interest after years of successful applications using first, second, and even third order high-resolution shock-capturing methods. In this study, we intend to investigate the effectiveness on accuracy, robustness, and efficiency, of combining the advection upstream splitting method (AUSM) [13], especially its recent all-speed version AUSM$^+$-UP [9], with two high order concepts, the weighted essentially non-oscillatory (WENO-JS) scheme [8] and the monotonicity preserving (MP) scheme [16]. Both use a high order interpolant for smooth flow and modify it when encountering discontinuities. Basically, WENO attempts to modify the interpolant by re-weighting sub-interpolants, and MP limits the value of interpolated data through criteria such as monotonicity and extrema preservation. We shall also include recent variants of the original WENO, WENO-M [7] and WENO-Z [2], for comparison.

The first step, reconstruction, provides two data sets obtained by interpolating solution data from cell center to cell edge locations in a directionally biased manner. The crucial requirement is to ensure that no extra oscillations are generated numerically; a strong statement is the so-called total variation diminishing (TVD) property put forth by Harten [5]. The two data sets, typically denoted as the "left" and "right" states, are the input to the evaluation of flux at the cell edges, formulated as the Riemann problem in a finite domain.

The second step, flux evaluation, perhaps contributing to the most diverse area of research in the upwind scheme, maps the state data to numerical fluxes that result in the spatial balance of fluxes through the volume faces. This step critically influences the solution accuracy and past studies have resulted in several prominent and widely-used flux schemes.

The last step, time integration, uses the numerical fluxes to advance the solution in time. A successful method for time advancement must promote accuracy, convergence, stability, efficiency, and ease of use. From an implementation standpoint, there are two major approaches for these types of methods: implicit and explicit. Implicit methods use numerical fluxes from time levels where the solution is unknown for advancement and require an efficient solution of a large matrix-vector equation. Explicit methods only use numerical fluxes from previous time levels for advancement and are calculated directly. The choice of time advancement strategy typically will have the most profound effect on the allowable range of time step size that maintains numerical stability. For our purposes, all included results are calculated using the explicit third order TVD Runge-Kutta method [15], denoted as RK3.

In Section 2, the well-known high order interpolation procedures of the weighted essentially non-oscillatory (WENO) method and the monotonicity preserving (MP) method are outlined. Afterwards, two strategies are outlined for reducing the order of the in-