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A New Higher Order Shear Deformation Model for Static Behavior of Functionally Graded Plates

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Abstract. In this paper, a new displacement based high-order shear deformation theory is introduced for the static response of functionally graded plate. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory presented is variationally consistent, has strong similarity with classical plate theory in many aspects, does not require shear correction factor, and gives rise to transverse shear stress variation such that the transverse shear stresses vary parabolically across the thickness satisfying shear stress free surface conditions. The mechanical properties of the plate are assumed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents. Numerical illustrations concerned flexural behavior of FG plates with Metal-Ceramic composition. Parametric studies are performed for varying ceramic volume fraction, volume fraction profiles, aspect ratios and length to thickness ratios. The validity of the present theory is investigated by comparing some of the present results with those of the classical, the first-order and the other higher-order theories. It can be concluded that the proposed theory is accurate and simple in solving the static behavior of functionally graded plates.

AMS subject classifications: 74K20

Key words: Functionally graded material, power law index, volume fraction, higher-order shear deformation theory, Navier solution.

1 Introduction

The concept of functionally graded materials (FGMs) were the first introduced in 1984 by a group of material scientists in Japan, as ultrahigh temperature resistant materials for air-

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craft, space vehicles and other engineering applications. Functionally graded materials (FGMs) are new composite materials in which the micro-structural details are spatially varied through non-uniform distribution of the reinforcement phase. This is achieved by using reinforcement with different properties, sizes and shapes, as well as by interchanging the role of reinforcement and matrix phase in a continuous manner. The result is a microstructure that produces continuous or smooth change on thermal and mechanical properties at the macroscopic or continuum level (Koizumi, 1993 [1]; Hirai and Chen, 1999 [2]). Now, FGMs are developed for general use as structural components in extremely high temperature environments. Therefore, it is important to study the wave propagation of functionally graded materials structures in terms of non-destructive evaluation and material characterization.

Several studies have been performed to analyze the mechanical or the thermal or the thermo-mechanical responses of FG plates and shells. A comprehensive review is done by Tanigawa (1995) [3]. Reddy (2000) [4] has analyzed the static behavior of functionally graded rectangular plates based on his third-order shear deformation plate theory. Cheng and Batra (2000) [5] have related the deflections of a simply supported FG polygonal plate given by the first-order shear deformation theory and third-order shear deformation theory to that of an equivalent homogeneous Kirchhoff plate [6]. The static response of FG plate has been investigated by Zenkour (2006) [7] using a generalized shear deformation theory. In a recent study, şimşek (2010) [8] has studied the dynamic deflections and the stresses of an FG simply-supported beam subjected to a moving mass by using Euler-Bernoulli, Timoshenko and the parabolic shear deformation beam theory. şimşek (2010) [9] Benchour et al. [10] and Abdelaziz et al. 2010 [11] studied the free vibration of FG beams having different boundary conditions using the classical, the first-order and different higher-order shear deformation beam and plate theories. The non-linear dynamic analysis of a FG beam with pinned-pinned supports due to a moving harmonic load has been examined by şimşek (2010) [12] using Timoshenko beam theory.

The primary objective of this paper is to present a general formulation for functionally graded plates (FGP) using a new higher order shear deformation plate theory with only four unknown functions. The present theory satisfies equilibrium conditions at the top and bottom faces of the plate without using shear correction factors. The hyperbolic function in terms of thickness coordinate is used in the displacement field to account for shear deformation. Governing equations are derived from the principle of minimum total potential energy. Navier solution is used to obtain the closed-form solutions for simply supported FG plates. To illustrate the accuracy of the present theory, the obtained results are compared with three-dimensional elasticity solutions [13] and results of the first-order and the other higher-order theories (Table 1).

In this study, a new displacement models for an analysis of simply supported FGM plates are proposed. The plates are made of an isotropic material with material properties varying in the thickness direction only. Analytical solutions for bending deflections of FGM plates are obtained. The governing equations are derived from the principle of minimum total potential energy. Numerical examples are presented to illustrate the ac-