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REVIEW ARTICLE

Reduced Basis Approximation and Error Bounds for Potential Flows in Parametrized Geometries

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Abstract. In this paper we consider (hierarchical, Lagrange) reduced basis approximation and *a posteriori* error estimation for potential flows in affinely parametrized geometries. We review the essential ingredients: i) a Galerkin projection onto a low-dimensional space associated with a smooth "parametric manifold" in order to get a dimension reduction; ii) an efficient and effective greedy sampling method for identification of optimal and numerically stable approximations to have a rapid convergence; iii) an *a posteriori* error estimation procedure: rigorous and sharp bounds for the linear-functional outputs of interest and over the potential solution or related quantities of interest like velocity and/or pressure; iv) an Offline-Online computational decomposition strategies to achieve a minimum *marginal computational cost* for high performance in the real-time and many-query (e.g., design and optimization) contexts. We present three illustrative results for inviscid potential flows in parametrized geometries representing a Venturi channel, a circular bend and an added mass problem.

AMS subject classifications: 65Y20, 76B99, 35Q35, 65N15

Key words: Reduced basis approximation, error bounds, potential flows, Galerkin method, a posteriori error estimation, parametrized geometries.

Contents

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1 Introduction and motivation

A great number of engineering problems require the solution of partial differential equations (PDEs) for many different configurations of the system. Even the computational costs for the solution of relatively simple parametrized problems may be very high and may remain unaffordable — although the computational power has increased considerably in the past few years. This makes necessary to develop techniques which are able to reduce the complexity of the system without a loss of information or accuracy of the results. The reduced basis method is a promising approach to fill this gap as it allows not only a *rapid* and *efficient*, but also a *reliable* solution of partial differential equations.

1.1 The input-output relation

In many applications, the main goal is not only the solution of the PDEs for the *field variables*, but the evaluation of *input-output relationships*. Here, the *output* is expressed as a functional of the field variables and can be for example an average quantity in the domain, an added mass or even a pointwise velocity and/or pressure. The *input-parameter* vector identifies a particular configuration of the system. Usually, this includes geometric variations, but also physical properties as well as boundary/initial conditions and sources. The *field variable* (as solution of the PDEs) connects the input parameters and the outputs of interest.

1.2 The many-query and real-time contexts

The reduced basis method allows us to reduce the *online* computational time (both of the field solution and of the outputs of interest) notably. This advantage is gained by additional *offline* effort. Therefore, the methodology presented in this work is suited particularly for problems arising in the *real-time context* or in the *many-query context*. For both these problem classes, the online performance is extremely important while increased offline effort is less critical and both are very challenging to the conventional solution