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On Spiral Waves Arising in Natural Systems

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Abstract. Spiral waves appear in many different natural contexts: excitable biological tissues, fungi and amoebae colonies, chemical reactions, growing crystals, fluids and gas eddies as well as in galaxies. While the existing theories explain the presence of spirals in terms of nonlinear parabolic equations, it is explored here the fact that self-sustained spiral wave regime is already present in the linear heat operator, in terms of integer Bessel functions of complex argument. Such solutions, even if commonly not discussed in the literature because diverging at spatial infinity, play a central role in the understanding of the universality of spiral process. In particular, we have studied how in nonlinear reaction-diffusion models the linear part of the equations determines the wave front appearance while nonlinearities are mandatory to cancel out the blowup of solutions. The spiral wave pattern still requires however at least two cross-reacting species to be physically realized. Biological implications of such a results are discussed.

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1 Introduction

Regular geometrical patterns occur in Nature in many situations [1], a striking case being as an example the observation of practically perfect spherical objects, neutron stars, as

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a consequence of self-gravitational general relativistic effects [2]. Another pattern very common is the spiral. It is remarkable the fact that D'Arcy Thompson, around one century ago, devoted an entire chapter of his classical monograph *"On Growth and Form"* to the appearance of the spiral form in Nature discussing in particular animal horns and molluscan shells [3]. Nowadays spiral waves have been observed in many other different biological contexts: in the heart, for example, the motion of the spiral center seems to be associated with specific types of arrhythmias [4], while in neural tissues this motion can be related to epilepsy and to spreading depressions in the retina [5]. More in detail these centers specifically are known as phase singularities, i.e., points in physical or abstract spaces near which the full cycle of isochrons crowds together. It is possible then to have a line of singularities, as in the singular filament of organizing centers, or along the border of a "black hole", i.e., a region on "latency diagrams" on which timings are lost [6]. These filaments in heart and brain tissues are non static and their motion in severe pathological states usually appears to be turbulent (what is it known as "chemical turbulence" [7]). Some biological populations of fungi and amoebae, like the *Dictyostelium discoideum*, tend to organize themselves in spiralling structures while spiral waves appear spontaneously also in specific chemical reactions like the classical Zhabotinsky-Belousov one [4] and also in growing crystals [8]. Common patterns encountered in all these systems are also target patterns, i.e., circular expanding waves generated by oscillatory local behaviors or external stimulations. Even in plant morphogenesis processes both these patterns can occur (kinetic phyllotaxis) [9]. In Fig. 1, a picture taken by one of the authors as an example, the bark of a dead tree manifests a spiralling pattern. The spiral is constituted by outer bark layers, which are well known to be associated with the early stages of the tree, so the typical arboreal radially diffusive behavior (a sort of target one-wave pattern) in this very peculiar case has been replaced by a spiralling mode, probably in association with a "very singular"event (a lightning, an infection or similar).

Figure 1: This picture of spiraling bark was taken in Rome from one of the authors on February, 2009.