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A CONJUGATE GRADIENT METHOD FOR DISCRETE–TIME OUTPUT FEEDBACK CONTROL DESIGN*

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Abstract

In this paper, the discrete–time static output feedback control design problem is considered. A nonlinear conjugate gradient method is analyzed and studied for solving an unconstrained matrix optimization problem that results from this optimal control problem. In addition, through certain parametrization to the optimization problem an initial stabilizing static output feedback gain matrix is not required to start the conjugate gradient method. Finally, the proposed algorithms are tested numerically through several test problems from the benchmark collection.

Mathematics subject classification: 49N35, 49N10, 93D52, 93D22, 65K05.
Key words: Output feedback control, Nonlinear conjugate gradient methods, Nonlinear programming.

1. Introduction

The static output feedback problem (SOF) for discrete or continuous-time control systems is one of the most studied problems, where wide area of applications in engineering and in finance are represented by this problem; see the two surveys [12,21] and the references therein. Particularly, many special purpose methods are designed by the engineers for solving this problem; see [12,21].

Various gradient-based methods are available for solving the SOF problem among them is the descent Anderson–Moore method [12] that solves the SOF problem by successfully minimizing particular quadratic approximation of the objective function combined with step-size rule. Mäkilä and Toivonen [12] solves the discrete problem by Newton's method with line search globalization. Rautert and Sachs [20] suggest quasi-Newton method with line search for solving the continuous-time SOF problem. Mostafa [16] introduces trust region method for solving the discrete-time SOF problem.

Levine–Athans method [12] is among the classical techniques for solving this problem. In this method a stationary point of the optimization problem is obtained by solving the system of the necessary optimality conditions under certain assumptions on the constant matrices of the problem. It has been reported that this method is computationally expensive and lacks of convergence properties.

All these methods are based on reformulating the discrete or continuous-time SOF problems into unconstrained matrix optimization problems. The formulation of the SOF problem as a constrained optimization problem allows utilizing numerous available constrained optimization techniques. Leibfritz and Mostafa [9] formulate the SOF problem as a nonlinear

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semi-definite programming problem and suggest for solving this problem an interior-point trust region method. Moreover, they suggest in [10] unconstrained and constrained trust region approaches for solving two formulations of the SOF problem. Kočvara et al. [7] consider the constrained formulation of the SOF problem and introduce an augmented Lagrangian semi-definite programming method. Mostafa [14] and [15] suggests a trust region method for solving the decentralized SOF problem and an augmented Lagrangian SQP method for solving a special class of nonlinear semi-definite programming problem related to the SOF problem, respectively.

In this paper, a nonlinear Conjugate Gradient (CG) method is analyzed and studied for solving the discrete-time SOF problem, which can be written as unconstrained optimization problem of the following form (see, e.g., the two surveys [12, 21]):

$$\min_{F \in \mathcal{S}_F} \quad J(F) = \mathrm{Tr}\,(P(F)Q(F)), \tag{1.1}$$

where the variable $P(F)$ is a matrix that solves the following discrete Lyapunov equation:

$$P(F) = A(F)P(F)A(F)^T + V, \tag{1.2}$$

$Q(F) = Q + C^T F^T RFC$, $A(F) = A + BFC$, and $\mathrm{Tr}\,(\cdot)$ is the trace operator. The variable F is a matrix that must be chosen from the following set of stabilizing output feedback controllers:

$$\mathcal{S}_F = \Big\{ F \in \mathbb{R}^{n_u \times n_y} \ : \ \rho\,(A + BFC) < 1 \Big\}, \tag{1.3}$$

where $\rho\,(\cdot)$ is the spectral radius. Moreover, A, B, C, Q, R, and V are given constant matrices of appropriate dimensions, which are defined and explained in Section 2. Problem (1.1)–(1.2) is an unconstrained optimization problem in the matrix variable F, where the eigenvalue condition $F \in \mathcal{S}_F$ will be fulfilled within the considered CG method.

Note, that the set $\mathcal{S}_F$ is open and in general unbounded. Therefore, it is convenient to define the following level set:

$$\mathcal{L}(F_0) = \{ F \in \mathcal{S}_F \ : \ J(F) \le J(F_0) \} . \tag{1.4}$$

This level set is compact; see [12, Appendix A]. For given $F_0 \in \mathcal{S}_F$ the theorem of Bolzano–Weierstrass ensures the existence of a unique solution to the optimization problem (1.1)–(1.2) in the level set $\mathcal{L}(F_0)$; see [12].

The CG method was proposed by Hestenes and Stiefel [6] early in 1952 for solving linear systems of algebraic equations. Fletcher and Reeves [5] in 1964 developed a CG method for solving unconstrained optimization problems. Moreover, many different CG methods have been proposed in recent years (see, e.g., [1, 3, 4, 11, 18] and the references therein).

There are many large and medium-scale applications in the literature of output feedback control design where higher order optimization methods fail to solve; see, e.g., the benchmark collection [8] for various engineering applications. The attempt in this paper is to apply a modified Dai-Yuan nonlinear CG method which belongs to the class of low storage methods for solving the problem (1.1)–(1.2). Moreover, the convergence theory given in [1] is extended to the considered algorithm.

The existence of an initial stabilizing SOF gain matrix $F_0 \in \mathcal{S}_F$ is one of the main obstacles that typically faces numerical methods that solve this problem class. By parameterizing the optimization problem the resulting CG method does not require initial $F_0 \in \mathcal{S}_F$ to start the iteration sequence. The modified algorithm is denoted by CG2.