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A MULTIVARIATE MULTIQUADRIC QUASI-INTERPOLATION WITH QUADRIC REPRODUCTION*

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Abstract

In this paper, by using multivariate divided differences to approximate the partial derivative and superposition, we extend the multivariate quasi-interpolation scheme based on dimension-splitting technique which can reproduce linear polynomials to the scheme quadric polynomials. Furthermore, we give the approximation error of the modified scheme. Our multivariate multiquadric quasi-interpolation scheme only requires information of location points but not that of the derivatives of approximated function. Finally, numerical experiments demonstrate that the approximation rate of our scheme is significantly improved which is consistent with the theoretical results.

Mathematics subject classification: 41A05, 41A25.
Key words: Quasi-interpolation, Multiquadric functions, Polynomial reproduction, $\mathcal{P}_n$-exact A-discretization of $\mathcal{D}^\alpha$, Approximation error.

1. Introduction

The approximation of multivariate functions from scattered data is an important theme in numerical mathematics. One of the methods to attack this problem is quasi-interpolation. For a set of functional values $\{f(X_j)\}_{1\le j\le n}$ taken on a set of nodes $\Xi = \{X_1, X_2, \cdots, X_n\} \subseteq \mathbb{R}^d$, the form of quasi-interpolation function $Q_f(X)$ corresponding to $f(X)$ is as follows

$$Q_f(X) = \sum_{j=1}^{n} f(X_j)\varphi_j(X), \tag{1.1}$$

where $\{\varphi_j(X)\}$ is a set of quasi-interpolation basis functions. Using quasi-interpolation there is no need to solve large algebraic systems. The approximation properties of quasi-interpolants in the case that X_j are the nodes of a uniform grid are well-understood. For example, the quasi-interpolant

$$\sum_{j=1}^{n} f(jh)\varphi\Big(\frac{X-hj}{h}\Big)$$

can be studied via the theory of principal shift-invariant spaces, which has been developed in several articles by de Boor et al. (see, e.g., [1,2]). Here φ is supposed to be a compactly

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supported or rapidly decaying function. Based on the Strang-Fix condition for φ, which is equivalent to polynomial reproduction, convergence and approximation orders for several classes of basis functions were obtained (see also [3-5]). Scattered data quasi-interpolation by functions, which reproduces polynomials, has been studied by Buhmann *et al.* [6], Dyn and Ron [7], Wu and Schaback [8], Feng and Li [9], Wu and Liu [10], and Wu and Xiong [11].

Beast and Powell [12] first proposed a univeriate quasi-interpolation formula where φ_i in (1) is a linear combination of the Hardy's MQ basis [13]

$$\phi_i(x) = \sqrt{(x-x_i)^2 + c^2}, \quad x, x_i \in \mathbb{R}$$

and low order polynomials. Their formula requires the derivative informations of f at the endpoints, which is not convenient for practical purposes. Wu and Schaback [8] proposed another quasi-interpolation formula with modifications at the endpoints. Wu-Schaback's formula is given by

$$\mathcal{L}_D f(x) = \sum_{i=0}^{n} f_i \alpha_i(x), \tag{1.2}$$

where $f_i, i = 0, \cdots, n$ are the values of $f(x)$ at nodes $\{x_i\}$ and the interpolation kernel $\alpha_i(x)$ is also formed from linear combinations of the MQ basis functions, plus a constant, and linear polynomial:

$$\alpha_0(x) = \frac{1}{2} + \frac{\phi_1(x) - (x - x_0)}{2(x_1 - x_0)}, \tag{1.3a}$$

$$\alpha_1(x) = \frac{\phi_2(x) - \phi_1(x)}{2(x_2 - x_1)} - \frac{\phi_1(x) - (x - x_0)}{2(x_1 - x_0)}, \tag{1.3b}$$

$$\alpha_i(x) = \frac{\phi_{i+1}(x) - \phi_i(x)}{2(x_{i+1} - x_i)} - \frac{\phi_i(x) - \phi_{i-1}(x)}{2(x_i - x_{i-1})}, \quad 2 \le i \le n-2, \tag{1.3c}$$

$$\alpha_{n-1}(x) = \frac{(x_n - x) - \phi_{n-1}(x)}{2(x_n - x_{n-1})} - \frac{\phi_{n-1}(x) - \phi_{n-2}(x)}{2(x_{n-1} - x_{n-2})}, \tag{1.3d}$$

$$\alpha_n(x) = \frac{1}{2} + \frac{\phi_{n-1}(x) - (x_n - x)}{2(x_n - x_{n-1})}. \tag{1.3e}$$

It is shown that (1.2) preserves monotonicity and convexity, and converges with a rate of $\mathcal{O}(h^{2.5} \log h)$ as $c = \mathcal{O}(h)$.

Ling [14] extended the univariate quasi-interpolation formula (1.2) to multidimensions using the dimension-splitting multiquadric basis function approach. Given data $\{(x_i, y_j, f_{ij}), i = 0, 1, \cdots, n, j = 0, 1, \cdots, m\}$, the form of dimension-splitting quasi-interpolation for MQ basis function is

$$\Phi_1 f(x, y) = \sum_{i=0}^{n} \sum_{j=0}^{m} f_{ij} \alpha_i(x) \beta_j(y), \tag{1.4}$$

where $\alpha_i(x), i = 0, 1, \cdots, n$ are given by (1.3). Along that y direction, the basis functions $\beta_j(y)$