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GALERKIN BOUNDARY NODE METHOD FOR EXTERIOR NEUMANN PROBLEMS*

Xiaolin Li
College of Mathematics Science, Chongqing Normal University, Chongqing 400047, China
Email: lxlmath@163.com
Jialin Zhu
College of Mathematics and Physics, Chongqing University, Chongqing 400044, China
Email: jlzhu@cqu.edu.cn

Abstract

In this paper, we present a meshless Galerkin scheme of boundary integral equations (BIEs), known as the Galerkin boundary node method (GBNM), for two-dimensional exterior Neumann problems that combines the moving least-squares (MLS) approximations and a variational formulation of BIEs. In this approach, boundary conditions can be implemented directly despite the MLS approximations lack the delta function property. Besides, the GBNM keeps the symmetry and positive definiteness of the variational problems. A rigorous error analysis and convergence study of the method is presented in Sobolev spaces. Numerical examples are also given to illustrate the capability of the method.

Mathematics subject classification: 65N12, 65N30, 65N38.
Key words: Meshless, Galerkin boundary node method, Boundary integral equations, Moving least-squares, Error estimate.

1. Introduction

Meshless (or meshfree) methods for numerical solutions of boundary value problems have attracted much attention in recent years [1, 2]. Compared to the finite element method (FEM) and the boundary element method (BEM), the main objective of this type of method is to get rid of, or at least to alleviate, the difficulty of meshing and remeshing the entire structure by simply adding or deleting nodes. Meshless methods have been found to have special advantages on problems to which the traditional mesh-based methods are difficult to be applied. These include problems with complicated boundary, moving boundary, large deformations, dynamic fracturing, mesh adaptively, and so on. Meshless methods can be categorized into domain type and boundary type. Several domain type meshless methods, such as the element free Galerkin method (EFGM) [1,3], the generalized FEM [4], the particle-partition of unity method [5], the h-p meshless method [6], the reproducing kernel particle method [2] and the finite point method [7] are very promising methods, and their mathematical backgrounds were well investigated.

Boundary integral equations (BIEs) are attractive computational techniques for linear and exterior problems as they can reduce the dimensionality of the original problem by one. Especially for exterior problems, the use of domain type methods requires discretization of the entire exterior, whereas with BIEs only the surface needs to be discretized. The boundary type meshless methods are developed by the combination of the meshless idea with BIEs, such as

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the boundary node method (BNM) [8], the boundary cloud method [9], the hybrid boundary node method [10], the boundary point interpolation method [11], the boundary element-free method [12] and the Galerkin boundary node method (GBNM) [13]. In contrast with the domain type methods, they are superior in treating problems dealing with infinite or semi-infinite domains. However, most boundary type meshless methods found in the literature lack a rich mathematical foundation to justify their use.

The BNM is formulated using the moving least-squares (MLS) approximations [1, 14] and the technique of BIEs. This method exploits the dimensionality of BIEs and the meshless attribute of the MLS. Nevertheless, since the MLS approximations lack the delta function property, the BNM cannot accurately satisfy boundary conditions. The strategy employed in the BNM [8] involves a new definition of the discrete norm used for the construction of the MLS approximations, which doubles the number of system equations.

The GBNM is a boundary type meshless Galerkin method which combines the MLS scheme and a variational formulation of BIEs. Compared with the BNM, boundary conditions in the GBNM can be satisfied directly and system matrices are symmetric. The main difference between the GBNM and the traditional Galerkin BEM is the way in which the shape function is formulated. In the GBNM, the boundary variables are approximated by the MLS technique that only use the boundary nodes, but in the Galerkin BEM, interpolants of the boundary variables are related to the geometry of the elements. The GBNM has been used for Dirichlet problems of Laplace equation [13] and biharmonic equation [15], and for Stokes flow [16]. In this paper, the GBNM is further developed for solving 2-D exterior Neumann problems.

As in many other meshless methods such as the EFGM and the BNM, background cells are used in the GBNM for numerical integration over the boundary. Cells are used for integration only, and have no restriction on shape or compatibility. The topology of cells can be much simpler than that of elements in the BEM or the FEM, since cells can be divided into smaller ones without affecting their neighbours in any way—such is not the case with boundary or finite elements. This feature makes meshless methods especially suited for adaptive procedures [17,18]. When there is no difference between the cell structure and the boundary, error estimates of the GBNM for solving Dirichlet problems have been established [13, 15]. Generally, as the element in the BEM, the cell structure is an approximation of the boundary. In the case of the cell structure is not coincided with the boundary, the error results from the approximation of the boundary by cells needs to be considered. In this paper, based on the preliminary error results of the GBNM for 2-D Dirichlet problems, we give an asymptotic error estimate of the GBNM for 2-D Neumann problems.

The following discussions begin with the brief description of the MLS approximation in Section 2. The formulations of the GBNM for exterior Neumann problems are developed in Section 3. Error estimates are established in Section 4. Section 5 presents regularization procedures for numerical integration. Section 6 provides some numerical results. Section 7 contains conclusions.

2. The Moving Least Squares Method

2.1. Notations

Let Ω be a bounded domain in $\mathbb{R}^2$ of points $\mathbf{x} = (x_1, x_2)$, its boundary Γ assumed to be sufficiently smooth, and let Ω' be the complementary of $\overline{\Omega} = \Omega + \Gamma$.