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A NON-OSCILLATORY KINETIC SCHEME FOR MULTI-COMPONENT FLOWS WITH THE EQUATION OF STATE FOR A STIFFENED GAS*

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Abstract

We extend the traditional kinetic scheme for ideal gases to the Euler equations with the equation of state for a multi-component stiffened gas. Based on a careful analysis of the oscillation mechanism of the traditional kinetic scheme across contact discontinuities, we propose a new non-oscillatory kinetic (NOK) scheme for multi-component stiffened gases. The basic idea in the construction is to use a flux splitting technique to construct numerical fluxes which do not depend on the concrete form of the equilibrium state. The new scheme can not only can avoid spurious oscillations of the pressure and velocity near a material interface which are observed in the traditional kinetic schemes such as the kinetic flux vector splitting (KFVS) and BGK schemes, but also can deal with the stiffened gas equation of state. Moreover, we also carry out a careful analysis on the consistency condition, truncation error and positivity of the NOK scheme. A number of 1D and 2D numerical tests are presented which demonstrate the accuracy and robustness of the new scheme in the simulation of problems with smooth, weak and strong shock wave regions.

Mathematics subject classification: 65M06, 82C40, 35L64, 76N10.
Key words: Kinetic scheme, Non-oscillation, Multi-component, Stiffened gases.

1. Introduction

In the past years, the development of numerical methods for compressible multi-component flows with general equations of state has attracted much attention. When a compressible inviscid flow includes several components, the flow can be modeled by the so-call extended Euler equations. The extended Euler equations consist of the traditional compressible Euler equations and the species equations which describe the conservation of the species. The extended Euler equations are no longer strictly hyperbolic, but a weakly hyperbolic system. And traditional conservative schemes, for example, the high order and high resolution schemes such as MUSCL [35], TVD [12], ENO [13] which work well in the numerical simulation of single component flows, may not give satisfactory numerical results for the extended Euler equations. In fact, it had been demonstrated in both theory and numerical tests that traditional conservative schemes could produce spurious pressure oscillations near material interfaces when they are used to solve the extended Euler equations [20].

Great efforts have been made to circumvent this difficulty in the past two decades [1, 11, 15, 20, 21, 30]. One of the popular methods is quasi-conservative algorithm proposed by Abgrall [1] and then generalized by Shyue [30–32]. Instead of the full conservative formulation, the quasi-conservative algorithm uses a quasi-conservative formulation of the extended Euler equations to

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ensure a consistent approximation of the energy equation near a material interface. A Godunov-type method with a generalized version of Roe's approximate Riemann solver was utilized to construct the quasi-conservative algorithm. The algorithm works well in the regions of smooth flows and moderate-strength shock waves. However, it was suggested by Shyue that it would be better to select the exact Riemann solver to deal with strong shock waves [30]. As is well-known, it is complex and expensive to construct the exact Riemann solver for general equations of state (EOS).

Instead, much efforts have been made in constructing Riemann-solver-free schemes of high-order and high-resolution, and one of such schemes is the gas kinetic scheme (GKS). Different from traditional Godunov-type methods, the GKS is based on the Boltzmann equation which provides more information on the flow and can describe the flux function of the governing equations by particle collision of the transport process. Once the particle distribution function on a cell interface is obtained, the numerical flux can be calculated directly.

In the last decades, significant progress has been achieved in the study of the GKS, see e.g., [6, 8, 9, 18, 19, 23, 34, 40], [24]– [28], [36]– [39]. The kinetic flux vector splitting scheme (KFVS) and the BGK scheme are among the GKS. A KFVS scheme solves the collisionless Boltzmann equation while a BGK scheme solves the Bhatnagar-Gross-Krook (BGK) model which is the most famous one of the simplified models of the Boltzmann equation. Besides, the so-called kinetically consistent difference scheme (KCDS) was proposed in 1980s by Russian mathematicians which in fact coincides with the KFVS (cf. [3]). Nowadays, the KCDS has been used in solving gas dynamical and quasi-gas dynamical problems, such as complex flows of viscous heat-conducting gases [10] and binary non-reacting gas mixture [9]. Also, the adaption of the KCDS to the architecture of multiprocessor systems with distributed memory was discussed in [3, Chapter IV].

In the recent years, the GKS has been used to simulate multi-component flows. The early attempt was made by Xu in 1997 [36], and then by Lian [22], Tang and Wu [33]. Further development can be found in [16, 17]. All the constructed schemes in [16, 17, 22, 33, 36] are based on the fully conservative formulation and work well when the difference of physical characters between two species is not too large. Consequently, these schemes cannot be applied to wider range of applications, in particular, to the simulation of flows with general EOS. In fact, two difficulties have to be overcome before the GKS can be widely used to simulate multi-components flow with general EOS. The first difficulty, as aforementioned, is that conservative schemes will produce pressure oscillations near a material interface. Such oscillations are also present in the conservative GKS. Furthermore, as pointed out by the authors [2], the traditional GKS will produce the pressure and velocity oscillations near contact discontinuities even in case of single material fluids. The second one is that the traditional GKS, which is based on the theory of rarefied gas dynamics, may not be extended directly to more general materials such as liquids and solids. In fact, it is difficult to construct a (universal) equilibrium state and a single kinetic transport equation to exactly recover the compressible Euler equations with general EOS of real materials.

The mechanism inducing the first difficulty was carefully analyzed by the authors of this paper [2] in the case of ideal gases, and a so-called consistent condition for constructing numerical fluxes was proposed to modify the traditional KFVS scheme, so that the oscillations across a contact discontinuity can be diminished. Consequently, a new modified scheme – MKFVS scheme was proposed to solve the quasi-conservative extended Euler equations. The new scheme eliminates spurious oscillations near a contact discontinuity in the case of both single and multi