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MULTISCALE ASYMPTOTIC EXPANSION FOR A CLASS OF HYPERBOLIC–PARABOLIC TYPE EQUATION WITH HIGHLY OSCILLATORY COEFFICIENTS*1)

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Abstract

In this paper, we will discuss the asymptotic behaviour for a class of hyperbolic -parabolic type equation with highly oscillatory coefficients arising from the strong-transient heat and mass transfer problems of composite media. A complete multiscale asymptotic expansion and its rigorous verification will be reported.

Key words: Multiscale asymptotic expansion, Hyperbolic-parabolic type equation, Highly oscillatory coefficient.

1. Introduction

Heat transfer theory has systematically been studied based on Fourier's law on theoretical analyses, numerical simulations and engineering applications. However, with rapid development of high-power laser technology, more and more interesting and surprising physical phenomena in strong-transient heat transfer process have been discovered(see, e.g. [13]). To interprete accurately these phenomena, non-Fourier's law was proposed (ref. [5]). It is interesting and important from both theoretical and practical points of view.

It is well known that , in the classical heat transfer theories, there exist some important tools, e.g. wave function expansion method, integral equation method and integral transform method and so on (see, e.g. [4,14] and references therein). Generally speaking, these analytic methods cannot be directly used to solve heat transfer problems of composite media, due to the complicated configurations and rapidly varying coefficients.

Clearly, being a fixed configuration, in principle it is always possible to choose the grid-size h small enough to reflect sufficiently local fluctuation of temperature functions.

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However, this is very often unpractical, since one would obtain linear systems having too many unknown (and actually it may be unfeasible in the higher dimensional case).

To overcome this principal difficulty, homogenization method has previously been analyzed for the above problems in some papers, see [2,6,9,12]. As we know , homogenization method based on average technique reflects only the global macroscopic properties of considering systems, and it is inadequate to describle the local changes of temperture functions. Thus, it is desirable to find out the macro- and meso-scale coupling formulations that can capture the effect of small scales as well as that of loading and constraints, which is an open problem presented by J.L.Lions in [9].

Our goal is to obtain the higher-order multiscale asymptotic expansion for a class of hyperbolic -parabolic type equation with rapidly oscillating coefficients.

The outline of this paper is organized in the following way. In section 2 , the physical model of strong-transient heat and mass transfer problems is introduced. In section 3, the corresponding mathematical equation and its solvablity are considered . In section 4 , we will obtain a multiscale asymptotic expansion for a class of hyperbolic-parabolic type equation with highly oscillatory coefficients in higher dimensional cases. ($n \geq 2$). Finally, we will complete the rigorous verification of the main theoretical result of this paper.

In the following, the Einstein summation convention is used: summation is taken over repeated indices. Throughout, C (with and without a subscript) denotes a generic positive constant, which is independent of ε unless otherwise stated.

2. Physical Model

It is well known that the energy equation of heat transfer problems is the following:

$$-(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z})+\dot{Q}=\rho c\frac{Du}{Dt} \tag{2.1}$$

where the heat flow density vector-valued function $\vec{q}=\{q_x,q_y,q_z\}$, and denotes by $\dot{Q}$ a thermal generation rate, ρ is a density function, c is a specific heat function, u is a temperature function.

If set $\omega_x=\frac{dx}{dt},\omega_y=\frac{dy}{dt},\omega_z=\frac{dz}{dt}$

$$\frac{Du}{Dt}=\frac{\partial u}{\partial t}+(\omega_x\frac{\partial u}{\partial x}+\omega_y\frac{\partial u}{\partial y}+\omega_z\frac{\partial u}{\partial z}) \tag{2.2}$$

$\frac{Du}{Dt}$ is called the material derivative, which it consists of two terms: the first one $\frac{\partial u}{\partial t}$ reflects local change of temperature, and the second one $(\omega_x\frac{\partial u}{\partial x}+\omega_y\frac{\partial u}{\partial y}+\omega_z\frac{\partial u}{\partial z})$ describes convection heat transfer. In particular, if we consider only the simple heat transfer process, then $\frac{Du}{Dt}$ reduces

$$\frac{Du}{Dt}=\frac{\partial u}{\partial t} \tag{2.3}$$

In contrast to classical Fourier's law, the non-Fourier's law needs to be added the delayed time term, i.e.

$$\vec{q}=-\vec{\lambda}\cdot grad\, u-\tau_0\frac{\partial q}{\partial t} \tag{2.4}$$