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APPLICATION OF NEWTON'S AND CHEBYSHEV'S METHODS TO PARALLEL FACTORIZATION OF POLYNOMIALS*1)

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Abstract

In this paper it is shown in two different ways that one of the family of parallel iterations to determine all real quadratic factors of polynomials presented in [12] is Newton's method applied to the special equation (1.7) below. Furthermore, we apply Chebyshev's method to (1.7) and obtain a new parallel iteration for factorization of polynomials. Finally, some properties of the parallel iterations are discussed.

Key words: Newton's method, Chebyshev's method, Parallel iteration, Factorization of polynomial.

1. Introduction

Let $F : R^N \to R^N$ be a nonlinear map. Newton's method

$$x^+ = x - F'(x)^{-1}F(x) \tag{1.1}$$

and Chebyshev's method

$$\begin{aligned}\widehat{x} &= x - [I + \tfrac{1}{2}F'(x)^{-1}F''(x)F'(x)^{-1}F(x)]^{-1}F'(x)^{-1}F(x)\\ &= x - F'(x)^{-1}F(x) - \tfrac{1}{2}F'(x)^{-1}F''(x)(F'(x)^{-1}F(x))^2\end{aligned} \tag{1.2}$$

are well known for solving nonlinear equation

$$F(x) = 0, \tag{1.3}$$

where I is the unit matrix of order N, x is an approximation of the solution x^* of (1.3), x^+ and $\widehat{x}$ are new approximations of x^* produced by Newton's and Chebyshev's methods, respectively. It is well known that the order of convergence for Newton's and Chebyshev's methods is 2 and 3, respectively, if $F'(x^*)$ is nonsingular.

Let

$$p(t) = \sum_{\nu=0}^{N} a_\nu t^{N-\nu},\ a_0 = 1 \tag{1.4}$$

be a monic polynomial of degree $N = 2n$. Then the convergence is quadratic or cubic, respectively, if Newton's or Chebyshev's method is used to find *one* simple zero of (1.4). Many parallel iterations have been proposed and studied to determine *all* zeros of (1.4) simultaneously (see [1]-[3], [5]-[11]).

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In the following it is assumed that $p(t)$ in (1.4) is a monic polynomial of degree $N = 2n$ with real coefficients. Then it can be factorized as

$$p(t) = \prod_{j=1}^{n}(t^2 - p_j t - q_j), \tag{1.5}$$

where $p_j, q_j (j = 1, 2, \cdots, n)$ are real.

Bairstow's method is a well known iteration to determine *one* real quadratic factor of $p(t)$ (see [4]). Its advantages are that the computational program is simple and that the convergence is quadratic if there are only simple or real double zeros of $p(t)$.

From the viewpoint of linear interpolation Zheng[12] proposed a family of parallel iterations to determine *all* real quadratic factors of polynomials, which keeps the advantages of Bairstow's method.

Let

$$g(t) = \prod_{j=1}^{n}(t^2 - u_j t - v_j) = \sum_{\nu=0}^{2n} b_\nu t^{2n-\nu}, b_0 = 1, \tag{1.6}$$

where $b_\nu = b_\nu(x)$ is the function of $x = (u_1, v_1, \cdots, u_n, v_n)^T \in R^{2n}$. It is clear that $(p_1, q_1, \cdots, p_n, q_n)^T$ of (1.5) is the solution of the system of nonlinear equations

$$F(x) = (f_1(x), \cdots, f_{2n}(x))^T = (b_1(x) - a_1, \cdots, b_{2n}(x) - a_{2n})^T = 0 \tag{1.7}$$

In section 4 of this paper it is shown in two different ways that one of the family in [12] is Newton's method applied to (1.7). In section 5 we apply Chebyshev's method to (1.7) and obtain a new parallel iteration for factorization of polynomials. For purpose of convenience the linear interpolation operators and their properties are introduced in section 2. A simple condition for nonsingulity of $F'(x)$ in (1.7) is given in section 3. Finally, some properties of the parallel iterations for factorization of polynomials are discussed in section 6.

2. Linear Interpolation Operators and Their Properties

For purposes of brevity, all formulas, sums and products involving indices i, j, k will assume the range $1, 2, \cdots, n$ and $\nu = 0, 1, \cdots, 2n$, unless explicit stated otherwise. We denote I and E the unit matrix of order $2n$ and 2, respectively. And 0 will denote real zero $0 \in R^1$, zero vector $(0,0)^T \in R^2$ or null matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, respectively, when it may not be mixed from context.

Definition [12]**.** *Let*

$$x_i = (u_i, v_i)^T \in R^2, \tag{2.1}$$

α_i, β_i be the zeros of

$$Q_i(t) = t^2 - u_i t - v_i. \tag{2.2}$$

Suppose that

$$L(f) = L(f; x_i, c; t) = l_1(f; x_i, c)(t - c) + l_2(f; x_i, c) \tag{2.3}$$

is the linear interpolation of $f(t)$ with nodes α_i, β_i, where $c \in R^1$ is a number independent of f and t. Denote

$$l(f; x_i, c) = (l_1(f; x_i, c), l_2(f; x_i, c))^T \in R^2, \tag{2.4}$$

$$A(f; x_i, c) = \begin{pmatrix} (u_i - 2c)l_1(f; x_i, c) + l_2(f; x_i, c) & l_1(f; x_i, c) \\ (v_i + u_i c - c^2)l_1(f; x_i, c) & l_2(f; x_i, c) \end{pmatrix}. \tag{2.5}$$

Particularly, we denote

$$L(f; x_i; t) = l(f; x_i, 0; t),$$