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A POSTERIORI ERROR ESTIMATES IN ADINI FINITE ELEMENT FOR EIGENVALUE PROBLEMS *1)

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Abstract

In this paper, we discuss a posteriori error estimates of the eigenvalue λ_h given by Adini nonconforming finite element. We give an assymptotically exact error estimator of the λ_h. We prove that the order of convergence of the λ_h is just 2 and the λ_h converge from below for sufficiently small h.

Key words: eigenvalue, nonconforming finite element, error estimate

Consider eigenvalue problems: Find pairs (λ, u), $\lambda \in R, u \in H_0^2(G), \|u\|_0 = 1$, such that

$$a(u,v) = \lambda(u,v), \quad \forall v \in H_0^2(G) \tag{1}$$

and their nonconforming finite element approximations:Find pairs $(\lambda_h, u_h), \lambda_h \in R, u_h \in V_h, \|u_h\|_0$=1, such that

$$a_h(u_h,v) = \lambda_h(u_h,v), \quad \forall v \in V_h \tag{2}$$

where $a(u,v) = \sum \int_G (a_{ijkl}\partial_i\partial_j u \partial_k \partial_l v + a_{pq}\partial_p u \partial_q v)$ is the symmetric, continuous, H_0^2-elliptic bilinear form, $(u,v) = \int_G uv; V_h$ is a nonconforming finit element space associated with a regular triangulations

$$T_h = \{T\},\ V_h \not\subset H_0^2(G),\ a_h(u,v) = \sum \sum_T \int_T (a_{ijkl}\partial_i\partial_j u \partial_k \partial_l v \ + \ a_{pq}\partial_p u \partial_q v)$$

are uniformly V_h -elliptic; i,j,k,l=1,2; p,q=0,1,2; $\partial_1 = \frac{\partial}{\partial x}$,$\partial_2 = \frac{\partial}{\partial y}$,$\partial_0 = id$, $\partial_1\partial_2 = \frac{\partial^2}{\partial x \partial y}$.

Let(λ_h, u_h) and (λ, u) be an eigenpair of (2) and of (1), respectivery, and (λ_h, u_h) converge (λ, u). In [3], the abstract error estimates has been presented and the following estimates has been proved for Adini finite element:

$$|\lambda_h - \lambda| \le Ch^2 \tag{3}$$

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In this paper ,we discuss a posteriori error estimates .We prove that the order of convergence is just 2, and give an asymptotically exact estimator for Adini finite element. Consider the steady state problems: Find $w \in H_0^2(G)$, such that

$$a(w,v) = (f,v), \quad \forall v \in H_0^2(G) \tag{4}$$

In the case of $f \equiv u_h$, let $u^\star$ *and* $u_h^\star \in V_h$ denote the exact solution and nonconforming finite element solution, respectively. It is obvious that $u_h^\star \equiv \lambda_h^{-1} u_h$.

Lemma 1. *The following estimates hold*

$$\frac{\lambda_h - \lambda}{\lambda} = \frac{\lambda_h}{(u,u_h)}(u^\star - u_h^\star, u) \tag{5}$$

$$\|u_h - u\|_s \le C\|u^\star - u_h^\star\|_s, \quad s = 0, 1 \tag{6}$$

Proof. Let P_λ be the orthogonal projection operator of the $L_2(G)$ onto eigenspace V_λ corresponding to the eigenvalue λ. Taking u=$\frac{P_\lambda u_h}{\|P_\lambda u_h\|_0}$.

$$\begin{aligned}(u^\star - u_h^\star, u) &= (u^\star - \lambda_h^{-1} u_h, u) = \lambda^{-1}(u_h,u) - \lambda_h^{-1}(u_h,u) \\ &= (\lambda^{-1} - \lambda_h^{-1})(u_h,u)\end{aligned}$$

which is just (5). The proof of the (6) is the same as that of [5, (1.4)].
In the case of $f \equiv \lambda u$,it is obvious that the exact solution of the associated (4) is just u and nonconforming finite element solution $u_h^0 \in V_h$ satisfies

$$a_h(u_h^0, v) = \lambda(u,v), \quad \forall v \in V_h \tag{7}$$

Lemma 2. *The following inequality holds*

$$\|u_h - u\|_h \le \|u_h^0 - u\|_h + C\|\lambda_h u_h - \lambda u\|_0 \tag{8}$$

Proof. From (2) and (7) we have

$$a_h(u_h - u_h^0, v) = (\lambda_h u_h - \lambda u, v)$$

Taking v=$u_h - u_h^0$, we get by uniformly elliptic

$$\begin{aligned}\|u_h - u_h^0\|_h^2 &\le Ca_h(u_h - u_h^0, u_h - u_h^0) \\ &\le C\|\lambda_h u_h - \lambda u\|_0 \|u_h - u_h^0\|_0\end{aligned}$$

and hence

$$\|u_h - u_h^0\|_h \le C\|\lambda_h u_h - \lambda u\|_0$$

using the above inequality and the triangle inequality we obtain (8).