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# PRIMAL PERTURBATION SIMPLEX ALGORITHMS FOR LINEAR PROGRAMMING*[1)]

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**Abstract**

In this paper, we propose two new perturbation simplex variants. Solving linear programming problems without introducing artificial variables, each of the two uses the dual pivot rule to achieve primal feasibility, and then the primal pivot rule to achieve optimality. The second algorithm, a modification of the first, is designed to handle highly degenerate problems more efficiently. Some interesting results concerning merit of the perturbation are established. Numerical results from preliminary tests are also reported.

*Key words*: Linear programming, Perturbation, Primal simplex algorithm, Partially revised tableau.

## 1. Introduction

Extensive research in linear programming, such as [1,2,9,10,11, 12,13,14,19], has been to improve pivot rules to reduce the number of iterations required. Relatively less effort was made on perturbing problem data with pivot rules unaltered (for instance, the self-dual parametric method [7] and perturbation-based methods [3,5]). And, because of the papametrization, the latter do not proceed as simply as the conventional simplex algorithm itself.

Recently, Pan [17] proposes new perturbation simplex algorithms that do not have such shortcoming, and performed favorably in computational tests. These algorithms can be regarded as the *dual* ones. The purpose of this paper is to develop *primal* algorithms of such type, and hence offer the other half of the approach.

Each of the proposed algorithms proceeds in a so-called 'partially' revised tableau form. First, some perturbed version of the original problem is solved. Then, a primally feasible tableau for the original problem is recovered from the optimal tableau of the perturbed problem. If optimality remains unchanged after the recovery, the original problem is already solved; otherwise, the recovered tableau is used as a starting point to achieve optimality.

In Section 2, we develop the Phase-1 procedure based on perturbation. In Section 3, we give some interesting theoretical results concerning this strategy. In Section 4, the procedure is combined with the simplex method to form a two-Phase algorithm for

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general purpose use. To solve highly degenerate problems more efficiently, the algorithm is then modified by applying the perturbation tactic to both phases in a cyclic manner. Finally, in Section 5, numerical results from our tests are reported, showing algorithms' promise of success.

## 2. The Perturbation Phase-1

We are concerned with linear programming problem in the standard form:

$$\max \quad z = cx \tag{2.1a}$$

$$\text{s.t. } Ax = b \tag{2.1b}$$

$$x \geq 0 \tag{2.1c}$$

where $A \in R^{m \times n}$ with $rank(A) = k \leq m < n$, $b \in R^m$, and $c$ and $x$ are row and column $n$-vectors, respectively.

Let an initial basis $B \in R^{m \times k}$ be given:

$$B = (a_{j_1}, \ldots, a_{j_k}), \tag{2.2}$$

where $a_{j_i}$ are the columns of $A$, corresponding to basic variables $x_{j_i} (i = 1, \ldots, k)$. Denote by $J_B$ the set of indices of basic variables and denote the remaining set by

$$\bar{J}_B = \{1, \ldots, n\} \backslash J_B. \tag{2.3}$$

Let $B^+$ be the Moore-Penrose inverse of $B$ and let $c_B$ be the price coefficients corresponding to basic variables. The following form, termed *partially revised* simplex tableau, will be useful to our purpose:

$$\begin{array}{c|c} \bar{c} & \bar{z} \\ \hline B^+A & \bar{b} \end{array} \tag{2.4}$$

where $\bar{z}$, $\bar{b}$ and $\bar{c}$ are determined by

$$\bar{z} = c_B B^+ b \tag{2.5a}$$

$$\bar{c} = c_B B^+ A - c \tag{2.5b}$$

$$\bar{b} = B^+ b \tag{2.5c}$$

In the case in which (2.4) is both primally and dually feasible, i.e., the following two sets

$$I = \{i \mid \bar{b}_i < 0. \ i = 1, \ldots, k\} \tag{2.6}$$

$$J = \{j \mid \bar{c}_j < 0, \ j \in \bar{J}_B\}, \tag{2.7}$$

are empty, the linear program is already solved, and hence we are done. Suppose that (2.4) is neither primally nor dually feasible. We perturb $\bar{c}_j (\forall j \in J)$ to some predetermined numbers $\delta_j > 0$, and consequently turn (2.4) into the following tableau:

$$\begin{array}{c|c} \bar{c}' & \bar{z} \\ \hline B^+A & \bar{b} \end{array} \tag{2.8a}$$

where

$$\bar{c}'_j = \begin{cases} \delta_j, & \forall j \in J, \\ \bar{c}_j, & \forall j \in \{1, \ldots, n\} \backslash J. \end{cases} \tag{2.8b}$$