Journal of Computational Mathematics, Vol.18, No.6, 2000, 609–620.

BOUNDING PYRAMIDS AND BOUNDING CONES FOR TRIANGULAR BÉZIER SURFACES*[1)]

Jian-song Deng   Fa-lai Chen
(*Department of Mathematics, University of Science and Technology of China, Hefei 230026, China*)

Li-li Wang
(*Information Center of Bengbu, Bengbu 233000, China*)

Abstract

This paper describes practical approaches on how to construct bounding pyramids and bounding cones for triangular Bézier surfaces. Examples are provided to illustrate the process of construction and comparison is made between various surface bounding volumes. Furthermore, as a starting point for the construction, we provide a way to compute hodographs of triangular Bézier surfaces and improve the algorithm for computing the bounding cone of a set of vectors.

Key words: Triangular Bézier surface patch, Hodograph, Bounding pyramid, Bounding cone.

1. Introduction

A *bounding pyramid/cone* of a rational Bézier surface patch is a pyramid/cone which has the property that if its vertex is translated to any point on the Bézier patch, the patch will lie completely outside the pyramid/cone. These kinds of pyramids/cones are useful tools in detecting closed loops in surface/surface intersections[2, 3] and determining directions for which a surface is single valued[5]. While methods of finding bounding pyramids and bounding cones for rectangular Bézier surfaces are widely addressed[2, 3, 4, 5, 7], no analogous results have ever been obtained for triangular surface patches. The purpose of this paper is to discuss the problem of computing bounding pyramids and bounding cones for triangular Bézier surfaces. Although the construction process presented in this paper shares some similarities with that for rectangular Bézier surfaces, it is still very valuable to fully describe the detailed process of the constructions due to the specialties of triangular Bézier surfaces.

The organization of this paper is as follows. We first provide an algorithm to compute the hodograph of a triangular Bézier surface and derive an upper bound for any partial derivative direction of the Bézier surface in Section 2. Then in Section 3, we present methods to compute bounding pyramid/cone of a set of vectors which are used to obtain the tangent bounding pyramids/cones of a Bézier surface in the

* Received December 2, 1997.
[1)]Supported by NKBRSF on Mathematics Mechanics (grant G1998030600), the National Natural Science Foundation of China (grants 69603009 and 19971087), the Research Fund for the Doctoral Program of Higher Education, a grant for Extinguished Young Teachers from Educational Committee of China, and Intramural Youth Science Foundation of USTC.

next section. Section 4 describes different approaches to construct surface bounding pyramids/cones based on the tangent bounding pyramids/cones, and comparison is made between these different bounding volumes. Finally in Section 5, an example is illustrated to demonstrate the whole construction process.

2. Hodographs of Triangular Bézier Surfaces

A triangular rational Bézier surfaces in homogeneous form is defined by

$$\begin{aligned}\mathbf{B}(\mathbf{P}) &:= \mathbf{B}(u,v,w) := (X(u,v,w), Y(u,v,w), Z(u,v,w), W(u,v,w)) \\ &:= \sum_{i+j+k=n} \mathbf{P}_{ijk} B^n_{ijk}(u,v,w)\end{aligned} \tag{1}$$

where $\mathbf{P}_{ijk} = (X_{ijk}, Y_{ijk}, Z_{ijk}, W_{ijk})$ are homogenous control points, and $B^n_{ijk}(u,v,w)$ are Bernstein bases with (u,v,w) being the barycentric coordinates of points $\mathbf{P}$ with respect to triangle domain $\mathbf{T} = \triangle \mathbf{T}_1\mathbf{T}_2\mathbf{T}_3$.

A direction in the domain plane can be expressed using barycentric coordinates as $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ with $\alpha_1 + \alpha_2 + \alpha_3 = 0$. For example, $\overrightarrow{\mathbf{T}_1\mathbf{T}_2} = (-1, 1, 0)$ and $\overrightarrow{\mathbf{T}_2\mathbf{T}_3} = (0, -1, 1)$.

In this section a single character in bold typeface signifies a homogenous point, while one with tilde denotes the corresponding Cartesian point or vector. For any two points $\mathbf{P}_i = (X_i, Y_i, Z_i, W_i), i = 1, 2$, we define[6]

$$Dir(\mathbf{P}_1, \mathbf{P}_2) = (W_1X_2 - W_2X_1, W_1Y_2 - W_2Y_1, W_1Z_2 - W_2Z_1) \tag{2}$$

'*Dir*' function indicates the direction of the Cartesian vector between two points, since

$$Dir(\mathbf{P}_1, \mathbf{P}_2) = W_1W_2(\tilde{\mathbf{P}}_1 - \tilde{\mathbf{P}}_2). \tag{3}$$

2.1 Derivative Directions of Triangular Rational Bézier Surfaces

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be any direction in domain plane. The derivative of $\mathbf{B}(\mathbf{P})$ along direction α is

$$\frac{\partial \mathbf{B}(\mathbf{P})}{\partial \alpha} = n \sum_{i+j+k=n-1} (\alpha_1 \mathbf{P}_{i+1,j,k} + \alpha_2 \mathbf{P}_{i,j+1,k} + \alpha_3 \mathbf{P}_{i,j,k+1}) B^{n-1}_{ijk}(u,v,w). \tag{4}$$

In Cartesian coordinates,

$$\frac{\partial}{\partial \alpha}\tilde{\mathbf{B}}(\mathbf{P}) = \frac{Dir(\mathbf{B}(\mathbf{P}), \frac{\partial}{\partial \alpha}\mathbf{B}(\mathbf{P}))}{W(u,v,w)^2}. \tag{5}$$

Thus the scaled hodograph $Dir(\mathbf{B}(\mathbf{P}), \frac{\partial}{\partial \alpha}\mathbf{B}(\mathbf{P}))$ gives the direction of $\frac{\partial}{\partial \alpha}\tilde{\mathbf{B}}(\mathbf{P})$.