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TWO ALGORITHMS FOR LC^1 UNCONSTRAINED OPTIMIZATION*[1)]

Wen-yu Sun
(*School of Mathematics and Computer Science, Nanjing Normal University, Nanjing 210097, China*)

R.J.B.de Sampaio
(*Applied Informatics Program, Pontifical Catholic University of Parana, Curitiba, PR, 81611-970, Brazil*)

Jin-Yun Yuan
(*Department of Mathematics, Federal University of Parana, Curitiba, PR, 81531-990, Brazil*)

Abstract

In this paper we present two algorithms for LC^1 unconstrained optimization problems which use the second order Dini upper directional derivative. These methods are simple and easy to perform. We discuss the related properties of the iteration function, and establish the global and superlinear convergence of our methods.

Key words: Nonsmooth optimization, Directional derivative, Newton-like method, Convergence, Trust region method.

1. Introduction

The LC^1 optimization problems exist extensively in various optimization problems. For example, the problems from nonlinear complementarity, variational inequality and C^2 nonlinear programming can be formed as LC^1 optimization problems. In addition, LC^1 optimization problems also arise from the extended linear-quadratic programming problems, nonlinear minimax problems, stochastic optimization problems and some semi-infinite programs. See [6] [9] [11] [13] [14] [15] [17] [21].

In this class of problems, the objective function is LC^1 function, but not a C^2 function, i.e., it is continuously differentiable and its derivative is only locally Lipschitzian but not necessary F-differentiable.

The Lipschitz condition plays a vital role in developing generalized Newton methods for solving nonsmooth equations. The research of the Lipschitz condition stimulates us to consider LC^1 opitimization and develop some superlinearly convergent methods for LC^1 optimization problems. Now several authors investigate LC^1 optimization problems. In [6] Hiriat-Urruty etc. considered the second order optimality condition for

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LC^1 optimization. In [11] Qi presented a superlinearly convergent approximate Newton's method for LC^1 optimization. In [16] we discussed generalized Newton's method for LC^1 unconstrained optimization which uses Clarke's generalized Hessian matrix. In [17] we gave a quasi-Newton-SQP method for general LC^1 constrained optimization in which the global convergence and superlinear convergence are established. A convergence result of BFGS method for LC^1 linearly constrained optimization is presented by Chen [2].

The development in this paper is closely related to Qi [11], Sun [16] and Sun [17], in which a similar theory is established for solving LC^1 optimization. As a continuation of our works, in this paper we consider using the second order Dini upper directional derivative and the trust region technique to deal with LC^1 optimization problem and present two methods. These methods are simple and easy to perform.

The organization of this paper is as follows. In the next section, we give some preliminaries which will be used in the whole paper. In Section 3, we set up our direction-finding subproblem for LC^1 optimization, discuss some related properties of the iteration function and give Algorithm 3.3. In Section 4, we establish the global convergence and superlinear convergence of Algorithms 3.3 for LC^1 optimization. In Section 5, we describe an approach by combining the Dini upper directional derivative and trust region technique, and establish its convergence properties.

2. Preliminaries

We first give some definitions that will be used for the remainder of the paper. In [16] we have defined the second order generalized directional derivative of f at x in the direction d as

$$f^{\circ\circ}(x;d) = \limsup_{x' \to x, t \downarrow 0} \frac{f^{\circ}(x'+td;d) - f^{\circ}(x';d)}{t}, \tag{2.1}$$

where $f^{\circ}(x;d)$ denotes the generalized directional derivative of f at x in the direction d. We also discussed some basic properties on $f^{\circ\circ}(x;d)$ in [16]. Now we consider

$$\min f(x),\ x \in R^n,\ f \in LC^1, \tag{2.2}$$

that means the objective function f we want to minimize is differentiable and ∇f is locally Lipschitzian. Therefore, the second order Dini upper directional derivative of the function f at $x_k \in R^n$ in the direction $d \in R^n$ can be defined to be

$$f''_D(x_k;d) = \limsup_{\lambda \downarrow 0} \frac{[\nabla f(x_k+\lambda d) - \nabla f(x_k)]^T d}{\lambda}. \tag{2.3}$$

If ∇f is directionally differentiable at x_k, we have

$$f''_D(x_k;d) = f''(x_k;d) = \lim_{\lambda \downarrow 0} \frac{[\nabla f(x_k+\lambda d) - \nabla f(x_k)]^T d}{\lambda} \tag{2.4}$$

for all $d \in R^n$. From [3] and [16], $f''_D(x_k;d)$ possesses the following basic properties:

1.
$$f''_D(x_k;\lambda d) = \lambda^2 f''_D(x_k;d) \tag{2.5}$$

and

$$f''_D(x_k;d_1+d_2) \le 2[f''_D(x_k;d_1) + f''_D(x_k;d_2)]. \tag{2.6}$$